

## A Proof of Eq.(18)

Equation (18) is

$$9L_n = 3L_{n-1} + 6M_{n-1}. \quad (1)$$

This can be derived by the straightforward calculation using the inverse of  $\mathbf{L}$ . But there is more easy method which uses the solution of

$$\begin{pmatrix} \Phi \\ \Phi \\ \Phi \end{pmatrix} = \begin{pmatrix} L & M & M \\ M & L & M \\ M & M & L \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}. \quad (2)$$

The solution of the above simultaneous equations is

$$\begin{pmatrix} I \\ I \\ I \end{pmatrix}, \quad (3)$$

where  $I = \Phi/(L + M + M)$ . Therefore, the summation of Eq.(2) is

$$3\Phi = (3L + 6M)I. \quad (4)$$

Using the above relation,

$$\Phi = \frac{3L + 6M}{9} 3I = \frac{3L + 6M}{9} \sum_{i=1,3} I_i \quad (5)$$

is obtained. Since  $3I$  is the current of the upper layer, the coefficient  $(3L + 6M)/9$  is the inductance of the upper layer. Hence, Eq. (18) is satisfied.