Mathematical Approach to Current Sharing Problem of Superconducting Triple Strands

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Introduction

- The current sharing between insulated strands in a superconducting cable is one of the important problems for its utilization.
- From the view points of the inverse problem, the sensitivity of current sharing between the insulated strands is determined by the condition number of the inductance matrix.
- In this work, we derive the formula to estimate the sensitivity of the current distribution against the displacement of inductance from the ideal case by use of the condition number.

Basic Equation of Current Sharing

$$\mathsf{L}\boldsymbol{I} = \Phi \boldsymbol{u},$$
$$\boldsymbol{u} \equiv \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix},$$
$$\Phi \equiv \int V(t)\mathrm{d}t.$$

L: Inductance Matrix
I: Current Vector of Wires

Although Φ is the function of time in general, we assume Φ is constant in this work because we need the ratio of current only.

Simplest Strands of Two Lines

$$\begin{pmatrix} L_1 & M \\ M & L_2 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \Phi \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\frac{I_1}{I_2} = \frac{L_2 - M}{L_1 - M}$$

The source of sharing is the dispersion of the self inductance.When the mutual inductance **M** is much smaller than the self inductance **L**, the current sharing is small even though the values of the self inductance are dispersed. Unfortunately, the small dispersion of **L** is amplified in the case of strands because of **M 〜 L**.

Simple Triple Strands (1)

$$\mathsf{L} = \begin{pmatrix} 1+\epsilon_1 & 1 & 1 \\ 1 & 1+\epsilon_1 & 1 \\ 1 & 1 & 1+\epsilon_2 \end{pmatrix}$$

$\varepsilon > 0$

$$\boldsymbol{I} = \begin{pmatrix} 1 \\ 1 \\ \frac{\epsilon_1}{\epsilon_2} \end{pmatrix}$$

- Here, we assume that all components of mutual inductance have the same value and one of the self inductances is changed barely.
- This solution shows all the components of current, which are distributed, have the same direction.

Simple Triple Strands (2)

$$\mathsf{L} = \begin{pmatrix} 1 & 1-\epsilon_1 & 1-\epsilon_2 \\ 1-\epsilon_1 & 1 & 1-\epsilon_1 \\ 1-\epsilon_2 & 1-\epsilon_1 & 1 \end{pmatrix}$$

$\epsilon \to 0$

$$\boldsymbol{I} = \begin{pmatrix} 1 \\ 2-\frac{\epsilon_2}{\epsilon_1} \\ 1 \end{pmatrix}$$

- We distribute the values of mutual inductances while the value of self inductance is fixed.
- This solution shows that the large dispersion of mutual inductance reverses the direction of current in a filament. Therefore, it is expected that the distribution of current sharing strongly depends on the dispersion of mutual inductance.

Simple Triple Strands (3)

Since the values of each element of the inductance matrix of the strands are close to each other, the solutions are undetermined in the case of ε =0 because the determinant of the matrixes is zero. Using arbitrary real parameters of a_i, the solution to this equation is expressed as

$$\boldsymbol{I} = \boldsymbol{x}_1 + a_2\boldsymbol{x}_2 + a_3\boldsymbol{x}_3$$

$\boldsymbol{x}_i$: eignevectors

$$\boldsymbol{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \boldsymbol{x}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \boldsymbol{x}_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

- All components of the current vector has the same value in the case of a_2=a_3=0 only.

General Strands

We expand the current vector $\boldsymbol{I}$ and the constant vector $\boldsymbol{u}$ by the eigenvectors $\boldsymbol{x}_i$ of the inductance matrix **L**.

$$\boldsymbol{I} = \sum_i a_i \boldsymbol{x}_i, \qquad \boldsymbol{u} = \sum_i b_i \boldsymbol{x}_i$$

$\boldsymbol{x}_i$: eignevectors

$$a_i = \frac{b_i}{\lambda_i}$$

λ_i:eigenvalues

- The components with small eigenvalues are amplified.

Condition Number

The solution to the simultaneous linear equations by matrix with small eigenvalues is sensitive to the change of coefficients, and called as **ill-posed problem**. Parameter that shows this sensitive nature is called the condition number.

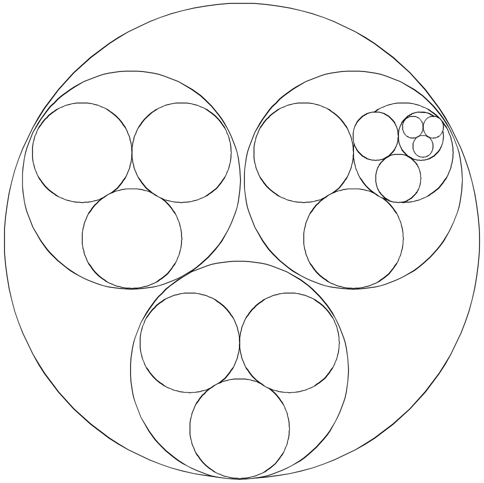
$$\mathrm{cond}(\mathsf{L}) = ||\mathsf{L}|| \cdot ||\mathsf{L}^{-1}|| = \frac{\lambda_{\max}}{\lambda_{\min}}$$

Here, $||\mathbf{L}||$ is norm of matrix $\mathbf{L}$ defined as the maximum eigenvalue. Using the condition number, the change of current caused by change of inductance is

$$\frac{|\delta \boldsymbol{I}|}{|\boldsymbol{I}|} \leq \mathrm{cond}(\mathsf{L}) \frac{||\delta \mathsf{L}||}{||\mathsf{L}||} = \frac{||\delta \mathsf{L}||}{\lambda_{\min}}$$

Multiple Triple Strands (1)

- Because the strand of level *n* consists of the strands of level *n*-1, the physical quantities on level *n* can be represented by those of subscript *n*.



$$M_n = (1-\epsilon_n)L_n \qquad (0 < \epsilon_n < 1)$$

$$\frac{L_n}{L_{n-1}} = 1 - \frac{2}{3}\epsilon_{n-1} \equiv \sigma_{n-1}$$

Schematic cross section

$$9L_n = 3L_{n-1} + 6M_{n-1}$$

$$\begin{aligned} L_n &= \sigma_{n-1}L_{n-1} = \cdots = L_0\Pi_{i=0}^{n-1}\sigma_i, \\ M_n &= (1-\epsilon_n)L_n = L_0(1-\epsilon_n)\Pi_{i=0}^{n-1}\sigma_i \end{aligned}$$

Multiple Triple Strands (2)

- Since the eigenvector with the maximum eigenvalue is ***u***, the solution to the multiple triple strands is the current vector whose all components have the same value.

$$\mathsf{L}_1 = \begin{pmatrix} L_0 & M_0 & M_0 \\ M_0 & L_0 & M_0 \\ M_0 & M_0 & L_0 \end{pmatrix}$$

$$\mathsf{M}_{n-1} = \begin{pmatrix} M_{n-1} & \cdots \\ \vdots & \ddots \end{pmatrix}$$

$$\lambda_{\max} = L_0 3^n \Pi_{i=0}^{n-1} \sigma_i$$

$$\lambda_{\min} = L_0 \epsilon_0$$

$$\mathsf{L}_n = \begin{pmatrix} \mathsf{L}_{n-1} & \mathsf{M}_{n-1} & \mathsf{M}_{n-1} \\ \mathsf{M}_{n-1} & \mathsf{L}_{n-1} & \mathsf{M}_{n-1} \\ \mathsf{M}_{n-1} & \mathsf{M}_{n-1} & \mathsf{L}_{n-1} \end{pmatrix}$$

$$\frac{|\delta \boldsymbol{I}|}{|\boldsymbol{I}|} \leq \frac{3^n \Pi_{i=0}^{n-1} \sigma_i}{\epsilon_0} \frac{||\delta \mathsf{L}_n||}{||\mathsf{L}_n||} = \frac{||\delta \mathsf{L}_n||}{\epsilon_0 L_0}$$

Multiple Triple Strands (3)

- When the deviation of inductance for each filament is almost equivalent, we can put the error matrix:

$$\delta \mathsf{L}_0 = \xi L_0 \begin{pmatrix} 1 & -1 & \cdots & -1 \\ -1 & 1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & 1 \end{pmatrix}$$

$$||\delta \mathsf{L}_n|| = 3^n \xi L_0$$

$$\frac{|\delta \boldsymbol{I}|}{|\boldsymbol{I}|} \leq 3^n \frac{\xi}{\epsilon_0}$$

The current sharing increases with the coupling between the filaments $1/\varepsilon_0$ and the number of nesting level n.

Example

R: Radius of Ring-Shaped Coil
a: Radius of Filament

$$L_0 = \mu_0 R\left(\log\frac{8R}{a} - \frac{7}{4}\right)$$

$$M_0 = \mu_0 R\left(\log\frac{8R}{2a} - 2\right)$$

$$\epsilon_0 = \frac{L_0 - M_0}{L_0} = \frac{\log 2 + \frac{1}{4}}{\log\frac{8R}{a} - \frac{7}{4}} \simeq 0.07$$

R=1m
a=1μm

The condition $\xi \sim 1\times 10^{-4}$ is required to suppress the error of current less than 10% at *n* =4.

Summary

- To investigate the current distribution of strands, the inductance matrix is analyzed using the technique of the inverse problem, which can avoid the numerical instability.
- The inductance matrix of the multiple triple strands is analytically derived using the self similarity, and the degree of current sharing against the dispersion of components of the inductance matrix is also obtained analytically.
- According to the analytic form of the degree of current sharing, it is shown that the degree increases with the number of layers and the magnetic coupling between the filaments.