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Design of Force-Balanced Coils for High Field Tokamak Reactors

H. Tsutsui,
S. Nomura, S. Tsuji-Iio, R. Shimada

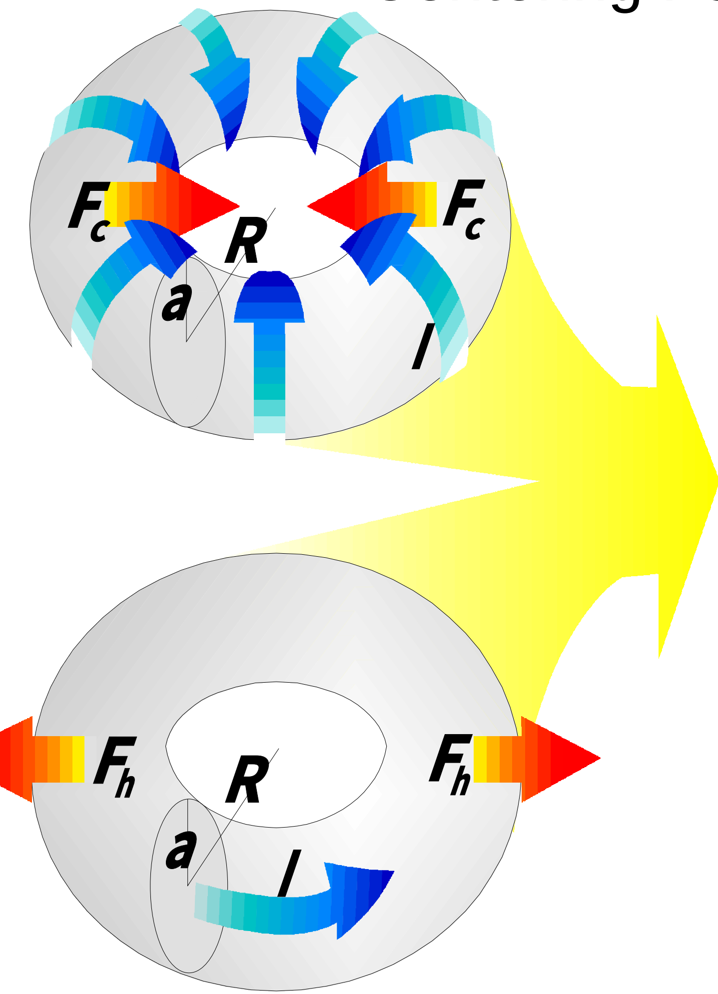
Research Laboratory for Nuclear Reactors,
Tokyo Institute of Technology

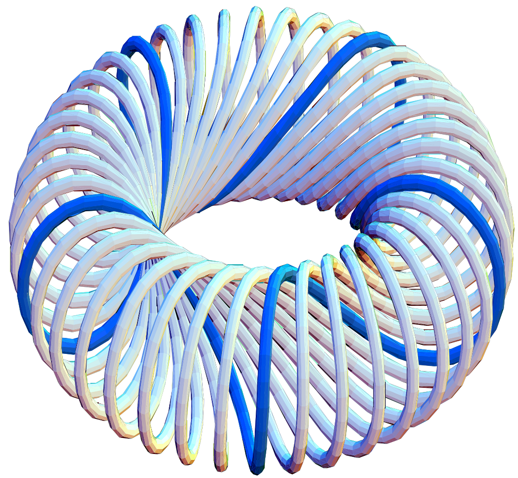
Introduction

- The **virial theorem** is the relation between the kinetic and the potential energies. The theorem, which is derived only form the equilibrium, shows that the tension is required to hold the magnetic energy.
- Using the **virial theorem,** we extended and generalized Force-Balanced Coil which is a helical type hybrid coil of the toroidal field (TF) coil and the solenoidal coil, and showed the condition to minimize the stress working in the coil (**virial-limit condition**).
- In this work, we extend our theory to arbitrary shape cross section, and try to optimize the shape of a cross section.

Principle of Force-Balanced Coil

Centering Force by Poloidal Current





Centering force is much reduced,
but
stress distribution is not investigated.

Force-Balanced Coil

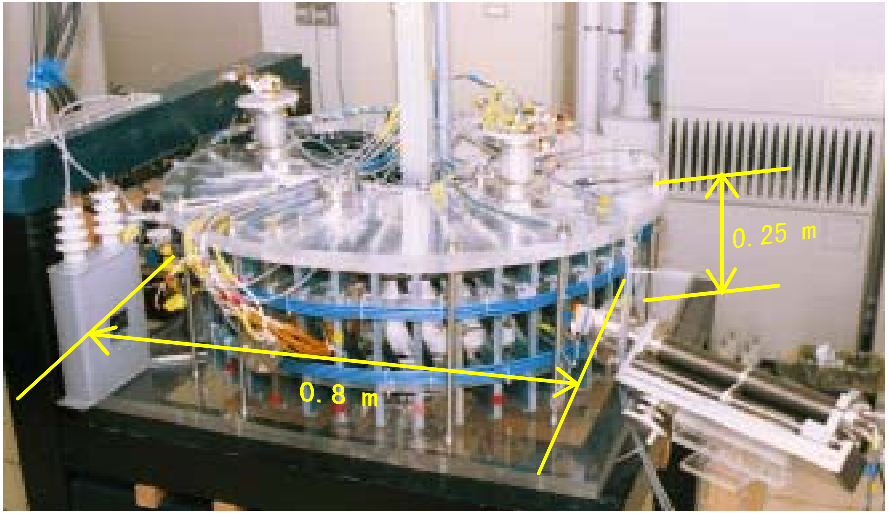
helical **hybrid** coil of **T**oroidal **F**ield **C**oils and **C**enter **S**olenoid

Hoop Force by Toroidal Current

Background

等々力(Equal Force)
TODOROKI-I

Parameter	Value
Toroidal Field	1T
Plasma Current	10kA
Time of Discharge	4ms



- The error field by FBC made the control of plasma difficult
- The force of toroidal direction was reduced in FBC. Is it held in stress ?

- Reduction Error Field
- Estimation of Stress
- Application of Virial Theorem

Virial Theorem

$$\boldsymbol{j}\times\boldsymbol{B}+\nabla\cdot\mathsf{S}=0$$

$$\nabla\times\boldsymbol{B}=\mu_0\boldsymbol{j}$$

$$\nabla\cdot\boldsymbol{B}=0$$

$\boldsymbol{j}$: current density

$\boldsymbol{B}$: magnetic field

S: stress tensor

Equilibrium Eq.

$$\nabla\cdot(\mathsf{T}+\mathsf{S})=0$$

$$\mathsf{T}\equiv\frac{1}{\mu_0}\left(\boldsymbol{BB}-\frac{B^2}{2}\mathsf{I}\right)$$

T: Maxwell stress tensor

$$\int \mathrm{Tr}(\mathsf{T}+\mathsf{S})\,\mathrm{d}V=0$$

$$\int\sum_{i=1}^{3}\sigma_i\,\mathrm{d}V=\int\frac{B^2}{2\mu_0}\mathrm{d}V\equiv U_{\mathrm{M}}>0$$

σ_i : principal stress

$$\tilde{\sigma}\equiv\frac{V_\Omega}{U_{\mathrm{M}}}\sigma$$

$$\langle\sigma\rangle\equiv\frac{\int\sigma\,\mathrm{d}V}{V_\Omega}$$

$$\left\langle\sum_{i=1}^{3}\tilde{\sigma}_i\right\rangle=1$$

- Positive stress (tension) is required to hold the field.
- Uniform tension is favorable.
- Theoretical limit is determined.

$$\tilde{\sigma}_1=\tilde{\sigma}_2=\tilde{\sigma}_3=\frac{1}{3}$$

Application to Thin Toroidal Shell

- We consider the toroidal coil with so large aspect ratio that toroidal effect is negligible.
- The current distribution is adopted which makes toroidal surface correspond to both current and magnetic surfaces.
- When torus is axisymmetric, the direction of principal stresses are ϕ and θ.

TF Coil (Major radius: *R*, minor radius: *a*, thickness: Δρ)

	ϕ	θ	Sumation
Stress	$-\frac{\mu_0 a^2 I_\theta^2}{16\pi^2 R\Delta\rho}$	$\frac{\mu_0 a^2 I_\theta^2}{8\pi^2 R\Delta\rho}$	
Integral	$-\frac{\mu_0 a^2 I_\theta^2}{4R}$	$\frac{\mu_0 a^2 I_\theta^2}{2R}$	$\frac{\mu_0 a^2 I_\theta^2}{4R}$
Energy			$\frac{\mu_0 a^2 I_\theta^2}{4R}$

Application to Helical Coil

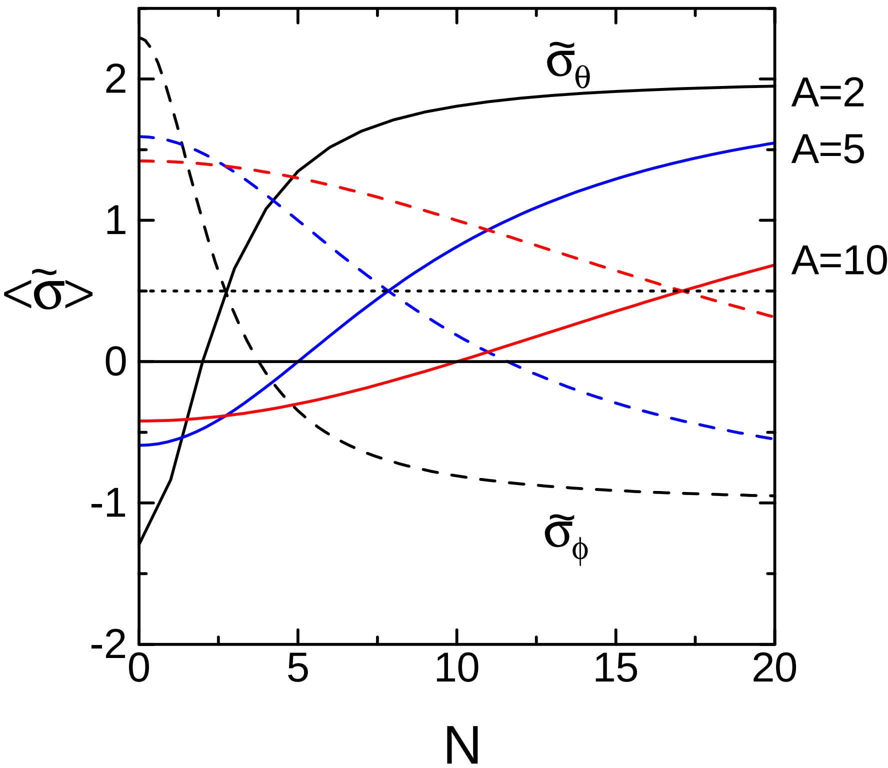
$$\left\langle \tilde{\sigma}_\theta \right\rangle = \frac{N^2 - A^2}{\dfrac{N^2}{2} + A^2 \log 8A - 2A^2}$$

$$\left\langle \tilde{\sigma}_\phi \right\rangle = \frac{A^2 \log 8A - A^2 - \dfrac{N^2}{2}}{\dfrac{N^2}{2} + A^2 \log 8A - 2A^2}$$

$$\left\langle \tilde{\sigma}_\theta \right\rangle + \left\langle \tilde{\sigma}_\phi \right\rangle = 1$$

$N \equiv \dfrac{I_\theta}{I_\phi}$: Pitch of Coil

A: Aspect Ratio

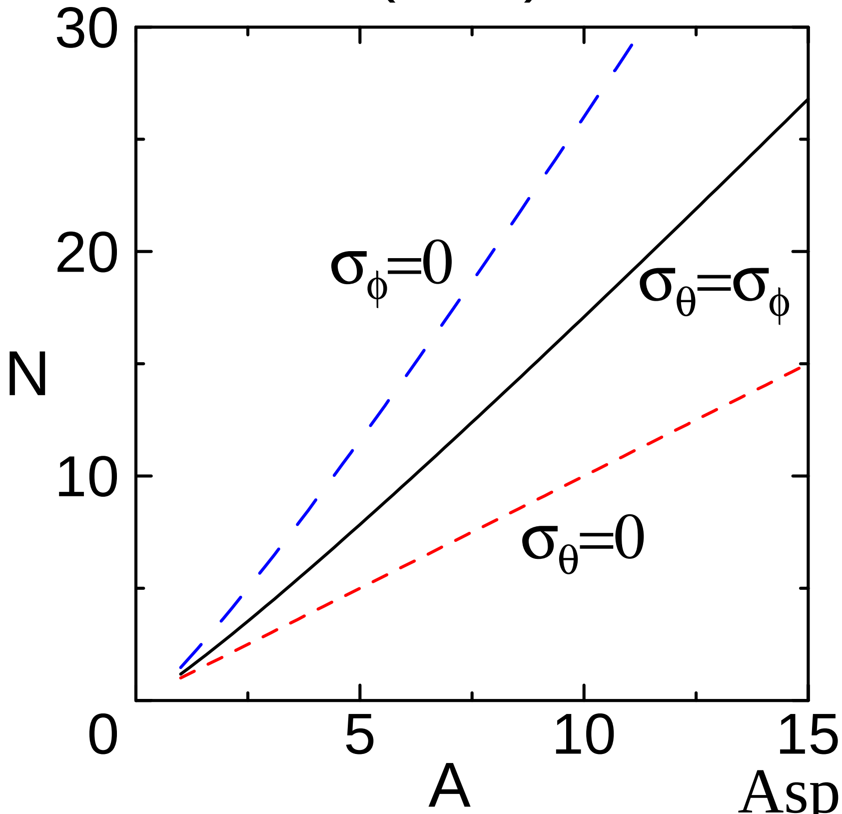


$\left\langle \tilde{\sigma}_\theta \right\rangle = \left\langle \tilde{\sigma}_\phi \right\rangle = \dfrac{1}{2}$ is optimal in energy.

Virial-Limit Condition

Shape of Coils

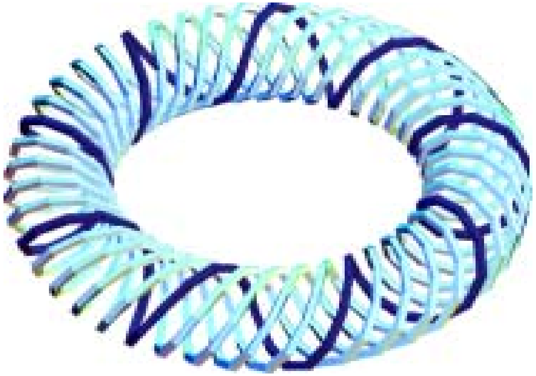
Relations of pitch number and aspect ratio of **Virial-Limit Coil** (**VLC**) etc.



$<\sigma_\phi>=0$

N=9

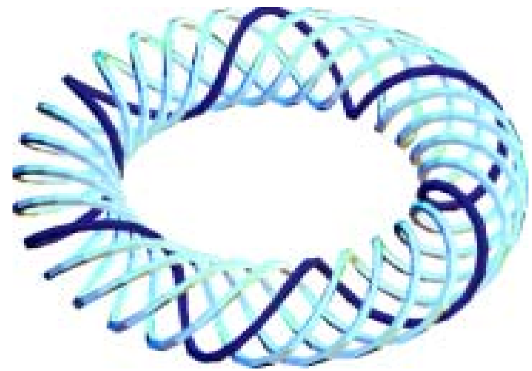
FBC



$<\sigma_\phi>=<\sigma_\theta>$

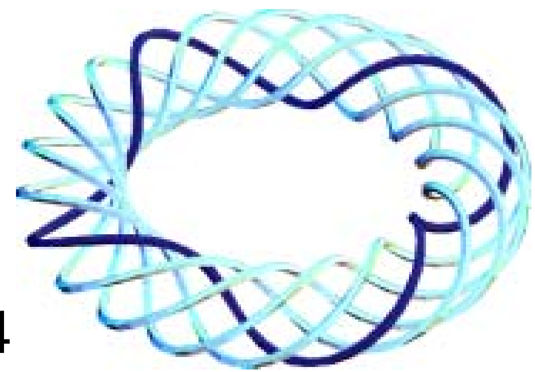
N=6

VLC



$<\sigma_\theta>=0$

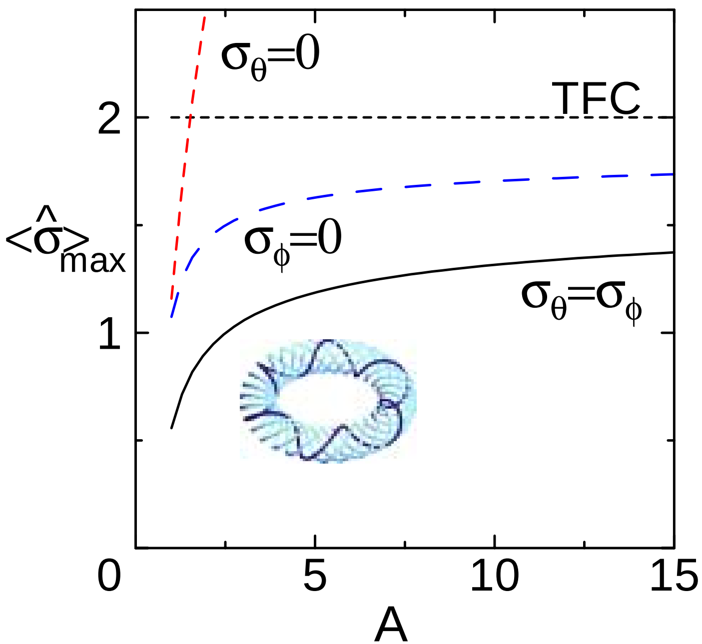
N=4



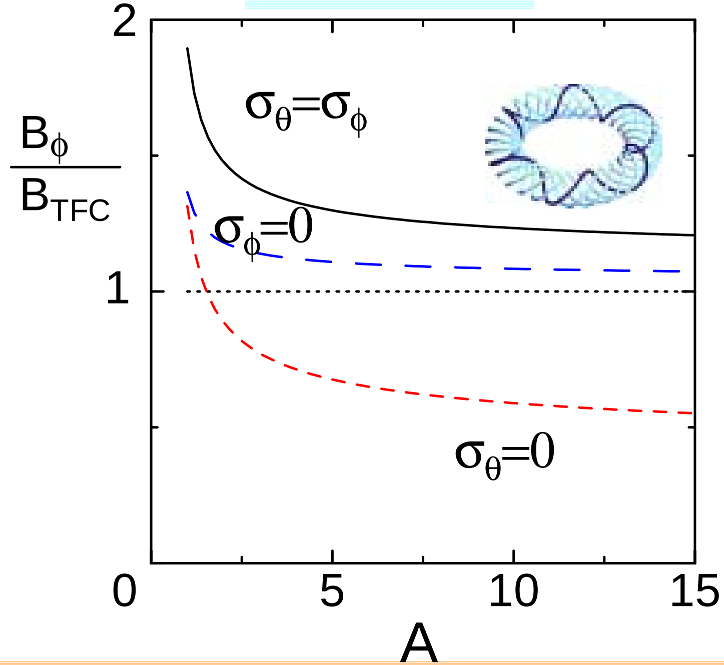
Aspect Ratio A =4

Comparison of Toroidal Field

$$\hat{\sigma} \equiv \frac{V_{\Omega}}{U_{TF}} \sigma$$



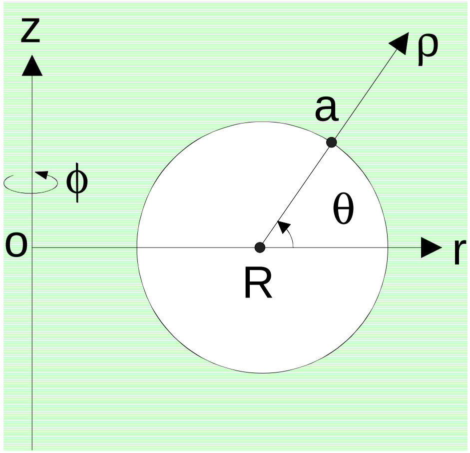
$$\frac{B_{\phi}}{B_{TFC}} = \sqrt{\frac{2}{\hat{\sigma}}}$$



- In the case of low aspect ratio, 1.5 times stronger magnetic field is created compared with traditional TF coil.

Toroidal Effect

Equilibrium of Magnetic Pressure and Stress



$$u(r) \equiv a\int_R^r r'\,p(r')\mathrm{d}r'$$

$$p \equiv \frac{B_\phi^2 - B_\theta^2}{2\mu_0}$$

$$T_\theta = \frac{u}{(r-R)r}$$

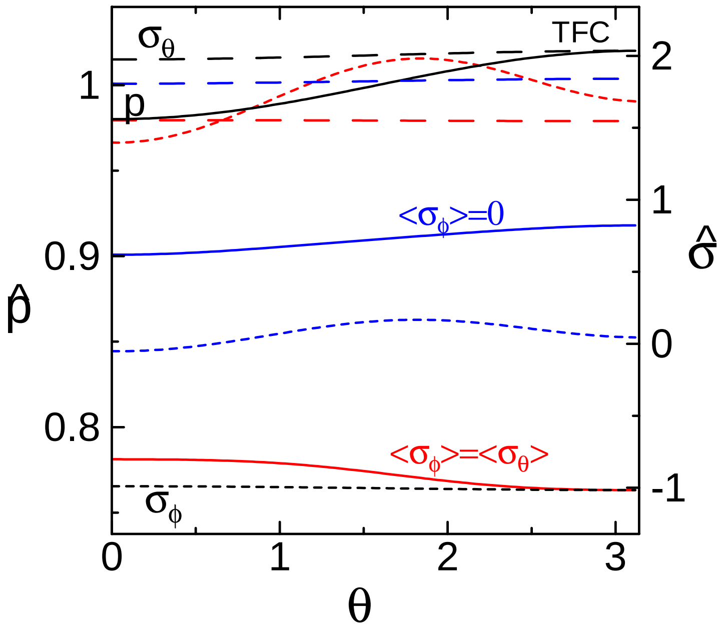
$$T_\phi = \frac{arp}{r-R} - \frac{u}{(r-R)^2}$$

- Distribution of stress in the toroidal shell with circular cross section is derived analytically by use of magnetic pressure.

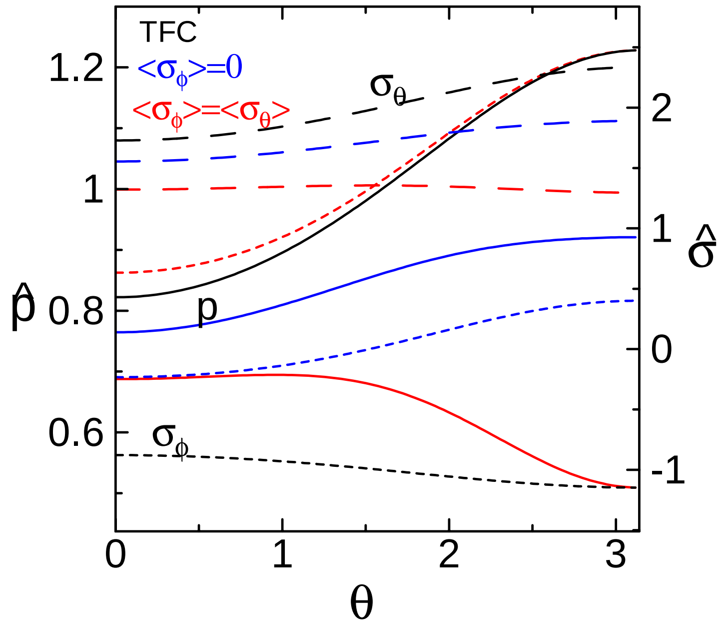
Distribution of Stress

(large aspect ratio)

A=100



A=10

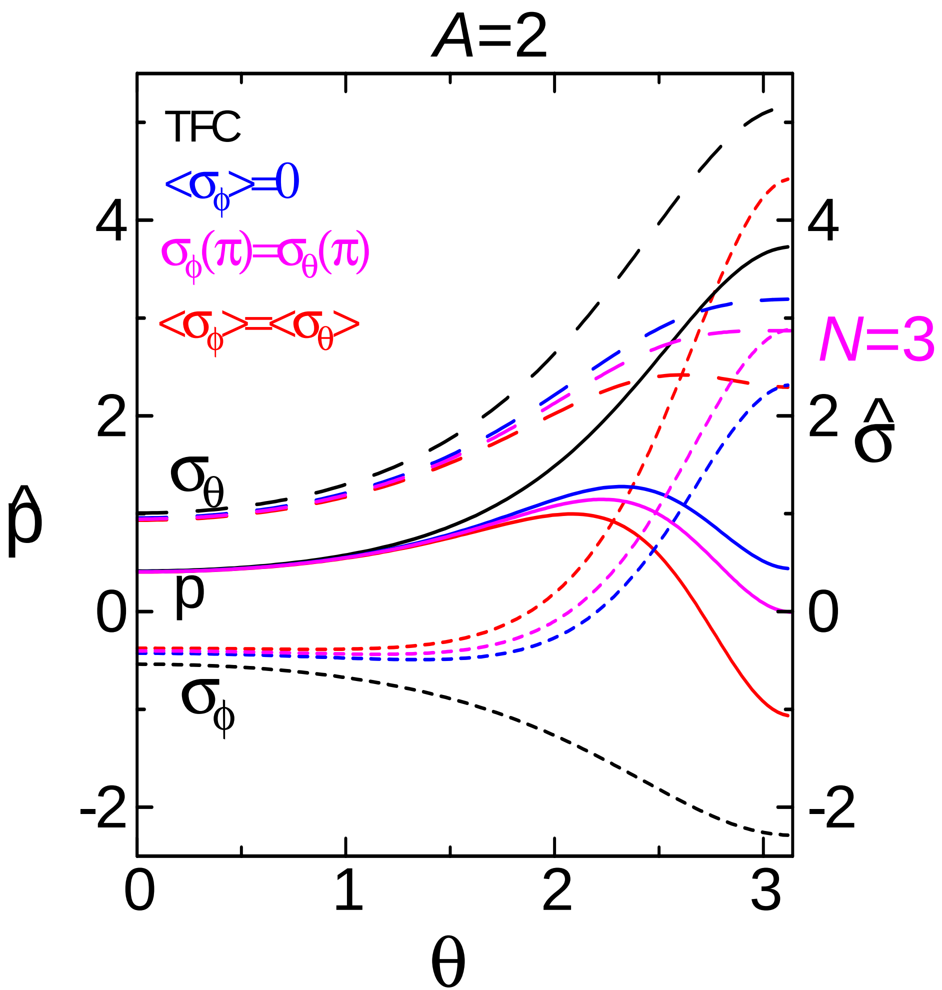


$$\hat{p} \equiv \frac{V_{\mathrm{T}}}{U_{\mathrm{M}}} \frac{B_\phi^2 - B_\theta^2}{2\mu_0}$$

- When A=100, distribution of stress is flat.
- There is no advantage of helical winding.

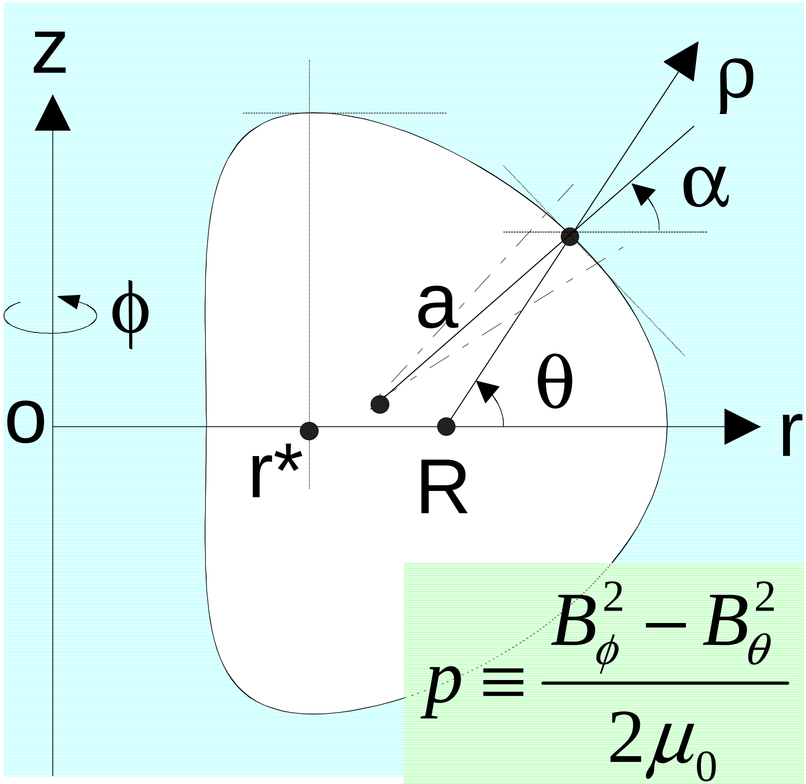
Distribution of Stress

(low aspect ratio)



- When A<10, distribution of stress is important.
- Assumption of large aspect ratio is not held.
- Optimal distribution is achieved to minimize the stress at $\theta=\pi$.

Non-Circular Cross Section



$$p \equiv \frac{B_\phi^2 - B_\theta^2}{2\mu_0}$$

$$u(r) \equiv \int_{r^*}^{r} r' p(r') \mathrm{d}r'$$

$$r^* \equiv r|_{\alpha=\pi/2}$$

$$T_\theta = \frac{u}{r\cos\alpha}$$

$$T_\phi = \frac{rp}{\cos\alpha} - \frac{u}{a\cos^2\alpha}$$

- The solution is only a modification in a case of a circular cross section.
- $R \rightarrow r^*$, $\theta \rightarrow \alpha$, $a \rightarrow a(r)$: curvature radius

2D Virial-Limit Coil

$$\frac{\mathrm{d}T_\theta}{\mathrm{d}r} = \frac{\mathrm{d}T_\phi}{\mathrm{d}r} = 0 \Rightarrow T_\theta = T_\phi \equiv T$$

$$r\frac{\mathrm{d}T_\theta}{\mathrm{d}r} + T_\theta = T_\phi = \frac{arp - rT_\theta}{a\cos\alpha}$$

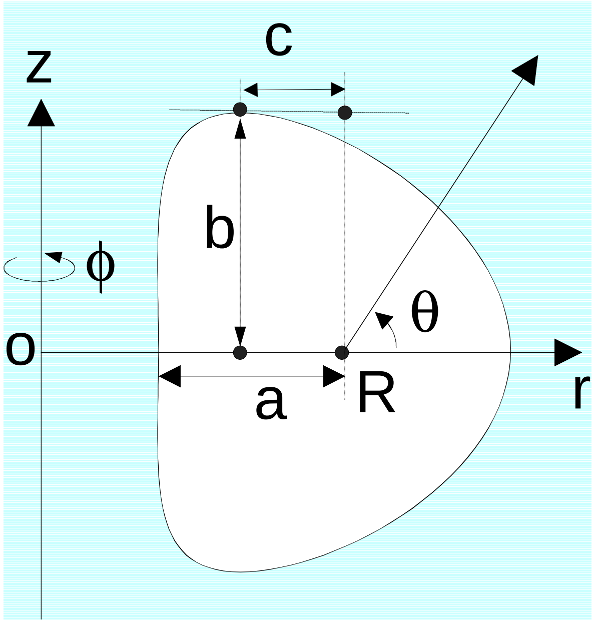
$$p = \frac{\mu_0}{8\pi^2} I_\phi^2 \left(\frac{N^2}{r^2} - \frac{f^2}{a^2} \right) = T\left(\frac{1}{a} + \frac{\cos\alpha}{r} \right)$$

$$N \equiv \frac{I_\theta}{I_\phi}, \quad \frac{1}{a} = \frac{\mathrm{d}\cos\alpha}{\mathrm{d}r}$$

$$j_\phi = I_\phi \frac{f(r)}{2\pi a}, \quad \oint \frac{f(r)}{2\pi a} \mathrm{d}s = 1$$

- f, a, cosα are functionals of $z(r)$ which expresses the shape of a cross section.
- Do solutions exist with any N and $z(r)$?

Searching Optimal Cross Section



$$\kappa \equiv \frac{b}{a}, \quad \delta \cong \frac{c}{a}$$

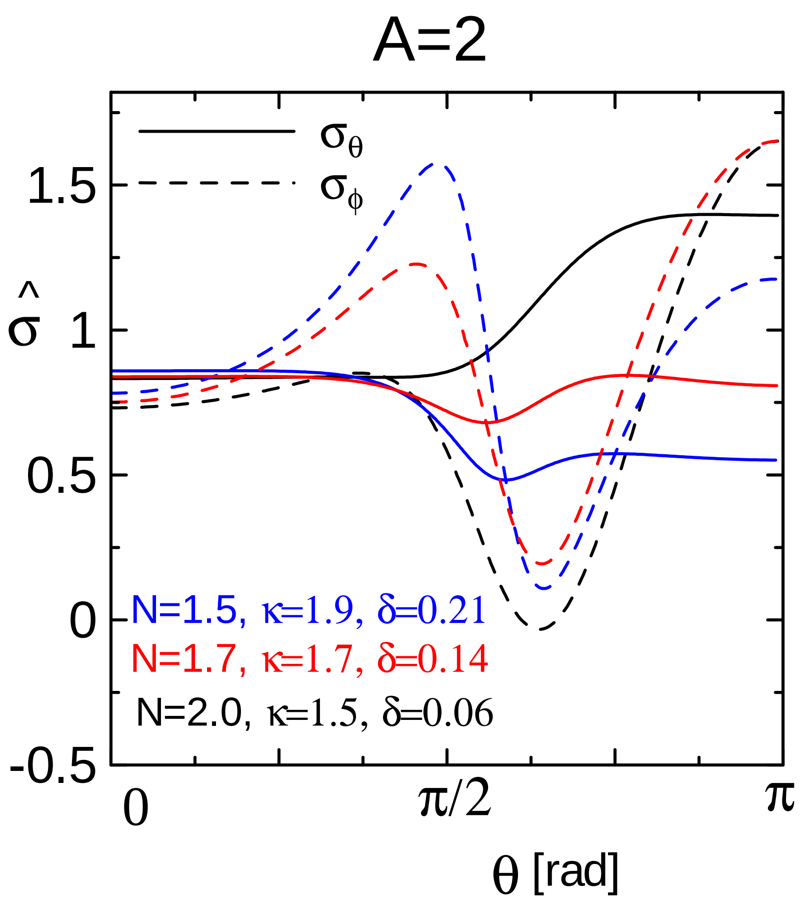
$$r = a\cos(\theta + \delta \sin\theta)$$

$$z = \kappa a \sin\theta$$

- Searching optimal cross section of $T_\theta = T_\phi$ with respect to κ, δ by Simplex method.

Distribution of Stress

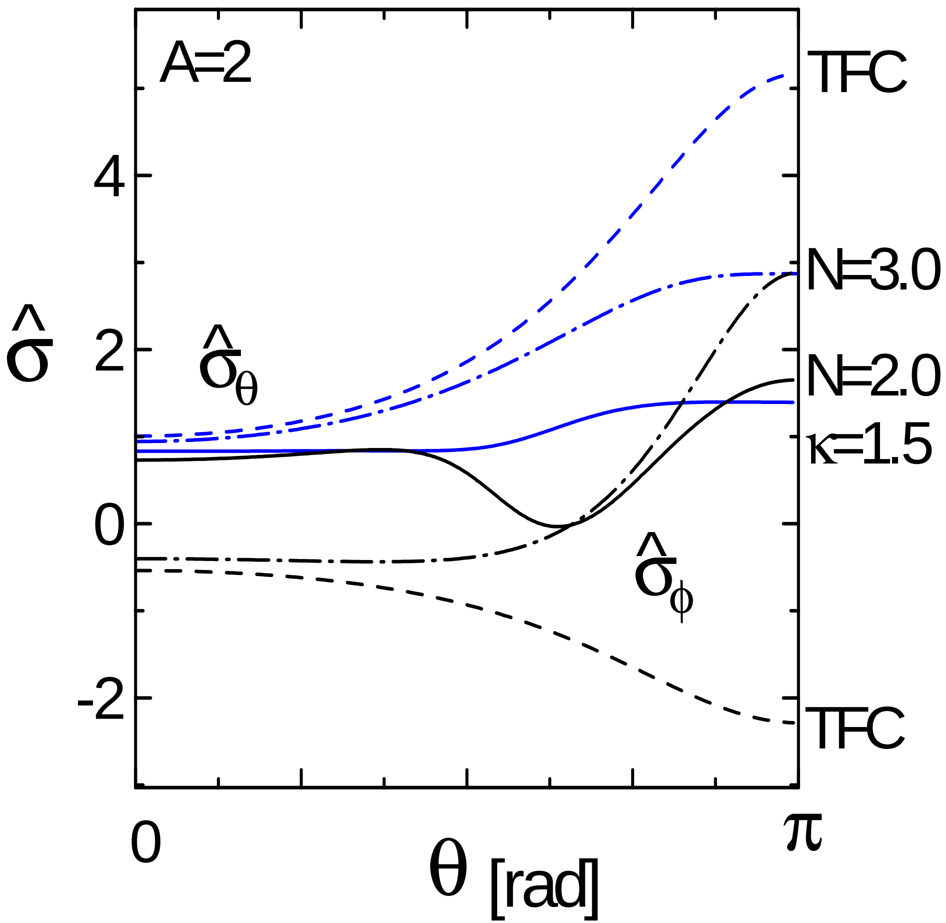
(non-circular)



- Searching cross section with flat stress distribution in A=2.
- Semi optimal cross sections with $1.5<\kappa<1.9$, $0<\delta<0.2$ are found in $1.5<N<2.0$.
- Maximum stress is reduced to about half (3.0→1.6) compared with that of circular cross section.

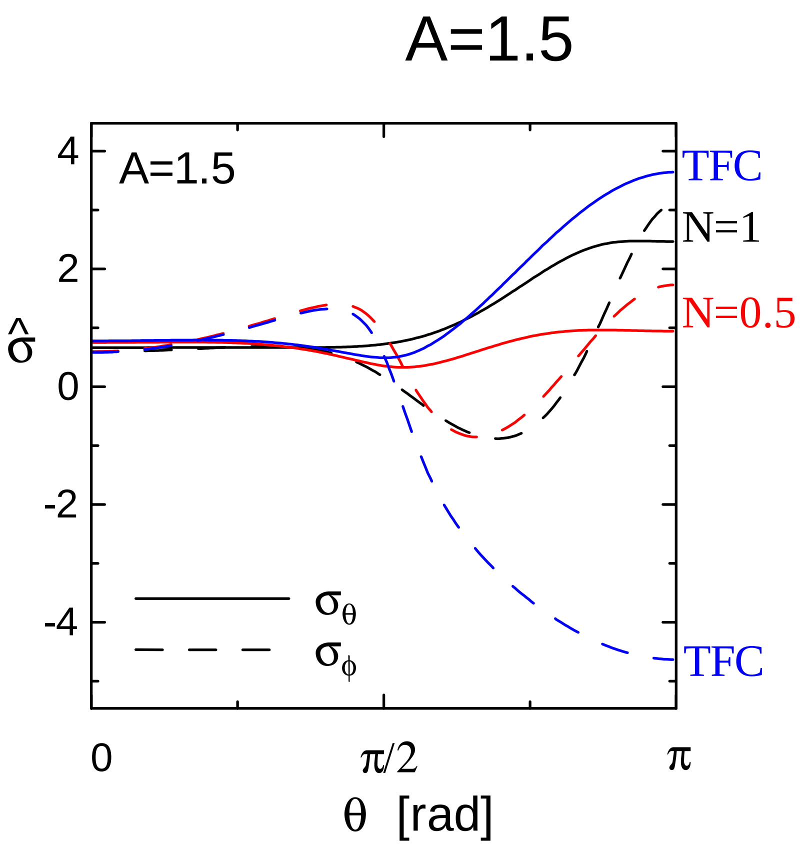
Distribution of Stress

(low aspect ratio)



- When $A<10$, distribution of stress is important.
- Assumption of large aspect ratio is not held.
- Optimal distribution is achieved to minimize the stress at $\theta=\pi$.

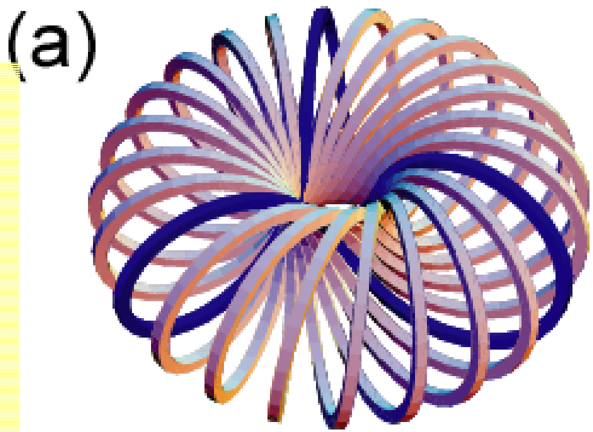
Distribution of Stress
(non-circular)



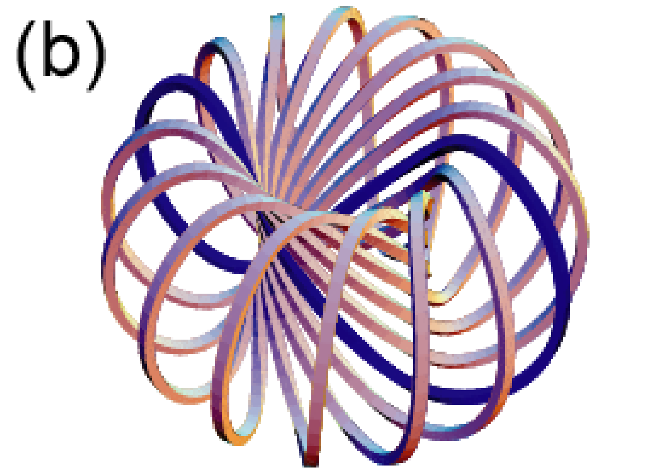
- Searching cross section with flat stress distribution in A=1.5.
- Semi optimal cross sections with 1.8<κ<3.3, 0<δ<0.4 are found in 0.5<*N*<1.0.
- Maximum stress is reduced to about half (3.6➔1.7) compared with that of the same cross section.

Shape of Virial-Limit Coils

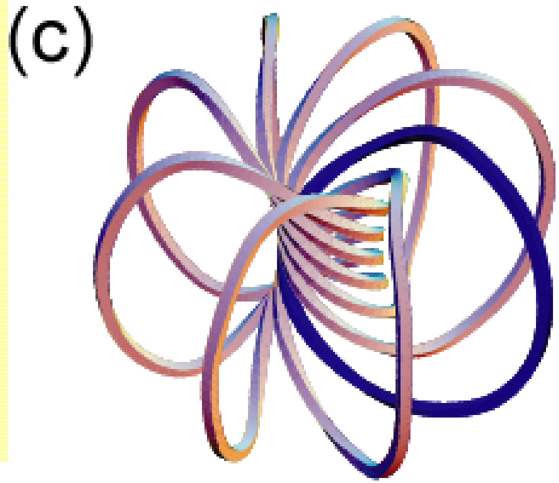
High elongation and low aspect ratio make directions of VLC winding become more vertical and horizontal in the outer and the inner sides of torus, respectively.



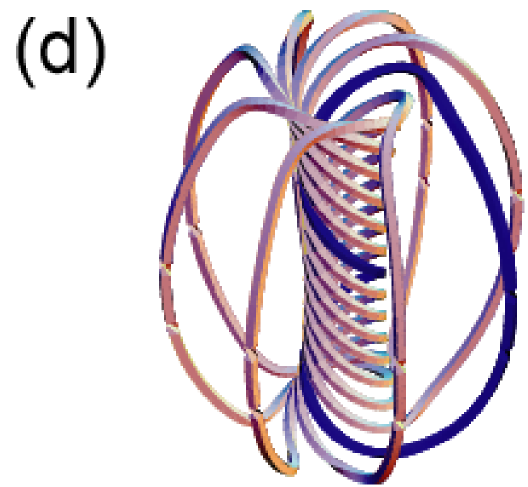
A=2, *N*=3, κ=1, δ=0



A=2, *N*=2, κ=1.5, δ=0



A=1.5, *N*=1, κ=1.8, δ=0



A=1.5, *N*=0.5, κ=3.3, δ=0

Summary

- The relation of toroidal field and stress is obtained by **virial theorem**, which shows that the optimal stress configuration is **uniform tensile stress**.
- **Shape optimization** of a poloidal cross section reduced the maximum stress to about half, and a virial-limit coil (**VLC**) makes 1.7 times stronger magnetic field than TF coils.
- Since the configuration of non-circular **VLC**s with high elongation and low aspect ratio is similar to that of CS and TF coil systems of conventional tokamaks, a VLC tokamak reactor can afford more room for blanket and use other parts in conventional tokamak reactors with much reduced volume of coils and their supporting structure.