A Tokamak with Nearly Uniform Coil Stress Based on Virial Theorem

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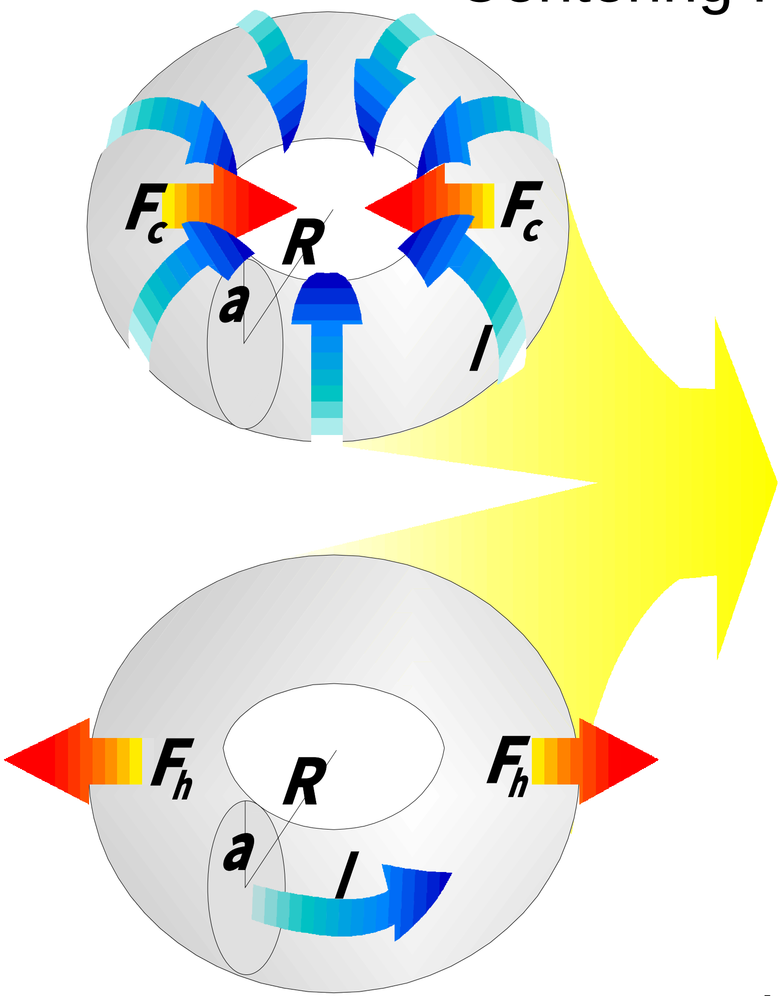
Introduction

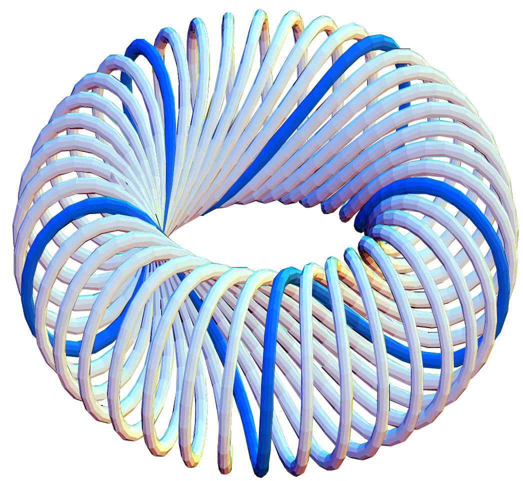
- We studied the tokamak with the Force-Balanced Coils which are hybrid helical coils of OH and TF coils and reduced the electromagnetic force.
- The virial theorem, which is derived only form the equilibrium, shows that the tension is required to hold the magnetic energy.
- The virial theorem in magnet systems is derived by the replacement of plasma pressure to stress.
- In this work, we extend the FBC by the virial theorem, and obtain the minimal stress condition.
- The new compact tokamak based on the virial theorem is designed and constructed.

Principle of Force-Balanced Coil

Centering Force by Poloidal Current



Hoop Force by Toroidal Current



Force-Balanced Coil

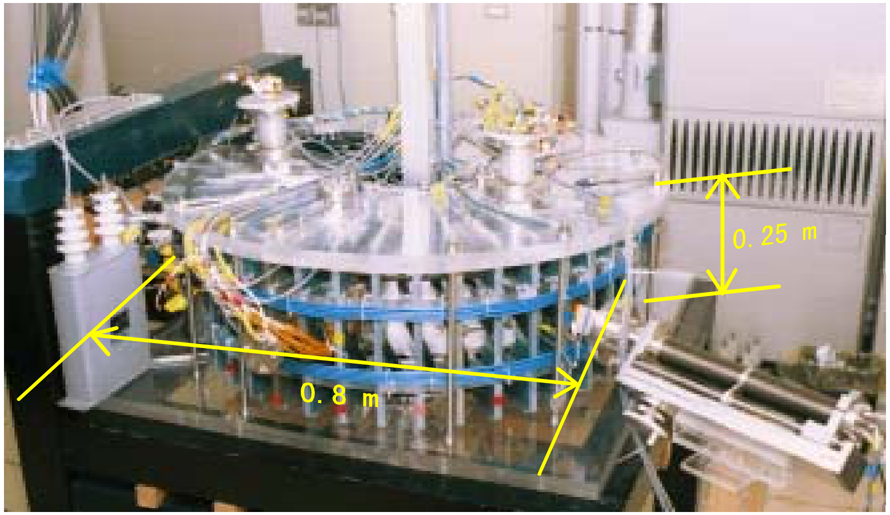
Centering force is much reduced,
but
stress distribution is not investigated.

Background

TODOROKI-I

Parameter	Value
Toroidal Field	1T
Plasma Current	10kA
Time of Discharge	4ms

- The error field by FBC made the control of plasma difficult.
- The force of toroidal direction was reduced in FBC. Is it held in stress ?



- Reduction Error Field
- Estimation of Stress
- Application of Virial Theorem

Virial Theorem

$$\boldsymbol{j}\times\boldsymbol{B}+\nabla\cdot\mathsf{S}=0$$
$$\nabla\times\boldsymbol{B}=\mu_0\boldsymbol{j}$$
$$\nabla\cdot\boldsymbol{B}=0$$

$\boldsymbol{j}$: current density

$\boldsymbol{B}$: magnetic field

S: stress tensor

Equilibrium Eq.

$$\nabla\cdot(\mathsf{T}+\mathsf{S})=0$$
$$\mathsf{T}\equiv\frac{1}{\mu_0}\left(\boldsymbol{BB}-\frac{B^2}{2}\mathsf{I}\right)$$

T: Maxwell stress tensor

$$\int\mathrm{Tr}(\mathsf{T}+\mathsf{S})\,\mathrm{d}V=0$$
$$\int\sum_{i=1}^{3}\sigma_i\mathrm{d}V=\int\frac{B^2}{2\mu_0}\mathrm{d}V\equiv U_\mathrm{M}>0$$

σ_i : principal stress

$$\tilde{\sigma}\equiv\frac{V_\Omega}{U_\mathrm{M}}\sigma$$
$$\langle\sigma\rangle\equiv\frac{\int\sigma\,\mathrm{d}V}{V_\Omega}$$

$$\left\langle\sum_{i=1}^{3}\tilde{\sigma}_i\right\rangle=1$$

- Positive stress (tension) is required to hold the field.
- Uniform tension is favorable.
- Theoretical limit is determined.

$$\tilde{\sigma}_1=\tilde{\sigma}_2=\tilde{\sigma}_3=\frac{1}{3}$$

Application to Helical Coil

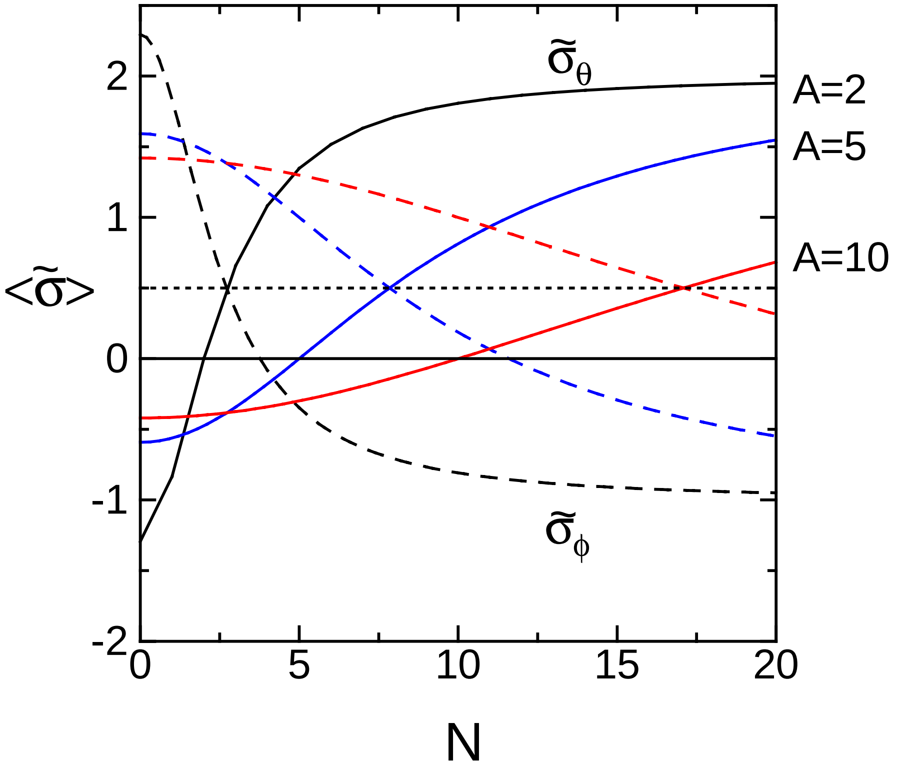
$$\left\langle \tilde{\sigma}_\theta \right\rangle = \frac{N^2 - A^2}{\frac{N^2}{2} + A^2 \log 8A - 2A^2}$$

$$\left\langle \tilde{\sigma}_\phi \right\rangle = \frac{A^2 \log 8A - A^2 - \frac{N^2}{2}}{\frac{N^2}{2} + A^2 \log 8A - 2A^2}$$

$$\left\langle \tilde{\sigma}_\theta \right\rangle + \left\langle \tilde{\sigma}_\phi \right\rangle = 1$$

$N \equiv \frac{I_\theta}{I_\phi}$: Pitch of Coil

A : Aspect Ratio

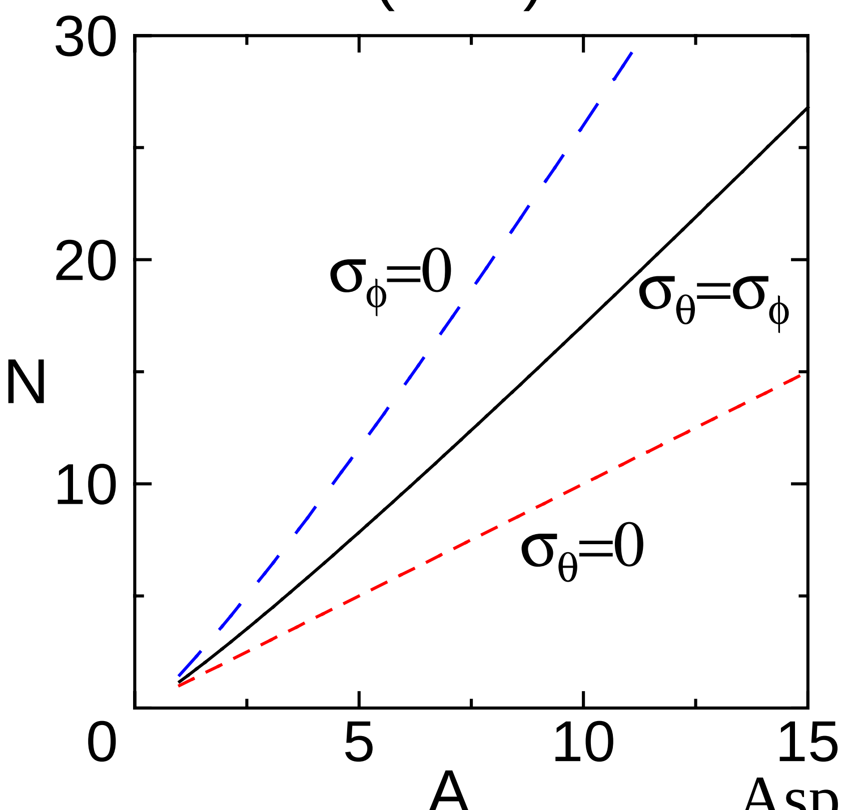


$\left\langle \tilde{\sigma}_\theta \right\rangle = \left\langle \tilde{\sigma}_\phi \right\rangle = \frac{1}{2}$ is optimal in energy.

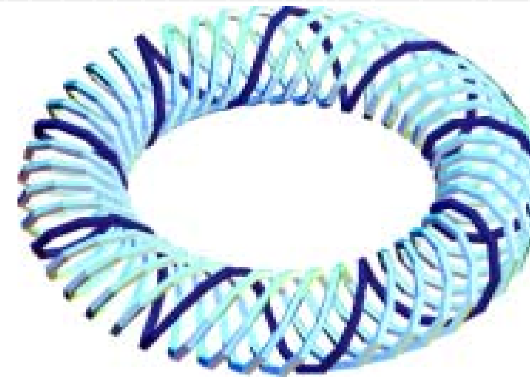
Virial-Limit Condition

Shape of Coils

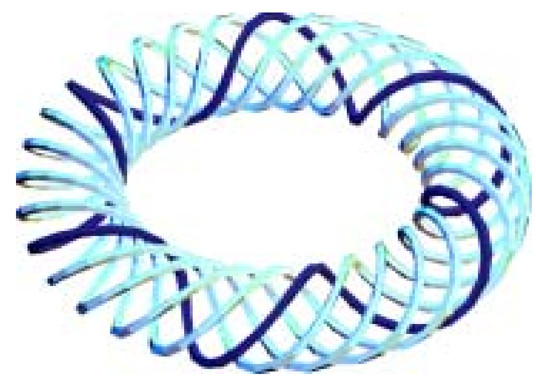
Relations of pitch number and aspect ratio of **Virial-Limit Coil** (**VLC**) etc.



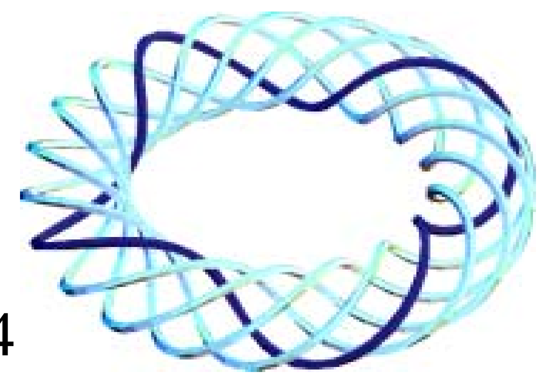
$\langle\sigma_\phi\rangle=0$
N=9
FBC



$\langle\sigma_\phi\rangle=\langle\sigma_\theta\rangle$
N=6
VLC



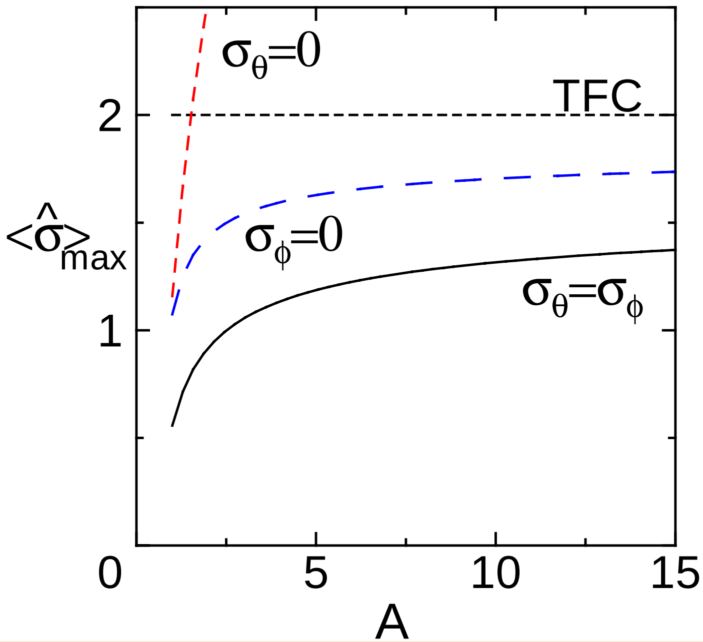
$\langle\sigma_\theta\rangle=0$
N=4



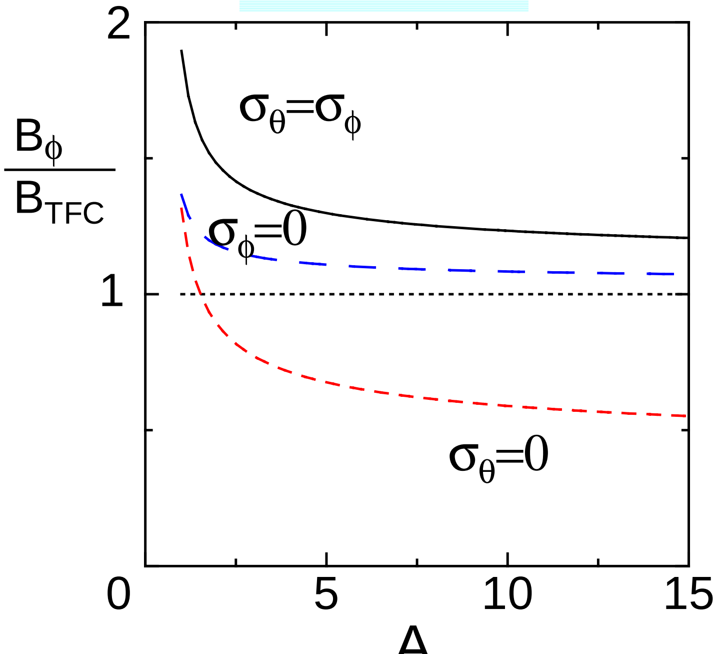
Aspect Ratio A =4

Comparison of Toroidal Field

$$\hat{\sigma} \equiv \frac{V_\Omega}{U_{\mathrm{TF}}}\sigma$$



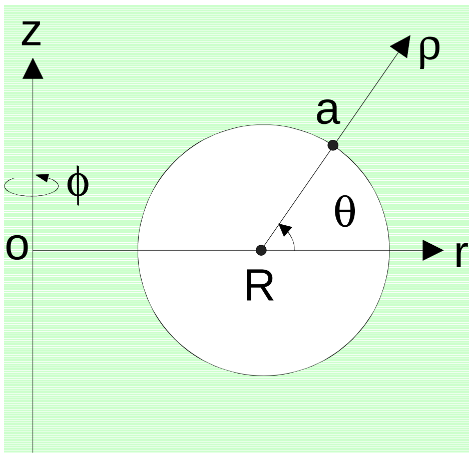
$$\frac{B_\phi}{B_{\mathrm{TFC}}} = \sqrt{\frac{2}{\hat{\sigma}}}$$



- In the case of low aspect ratio, 1.5 times stronger magnetic field is created compared with traditional TF coil.

Toroidal Effect

Equilibrium of Magnetic Pressure and Stress



$$u(r) \equiv a\int_R^r r'\, p(r')\mathrm{d}r'$$

$$p \equiv \frac{B_\phi^2 - B_\theta^2}{2\mu_0}$$

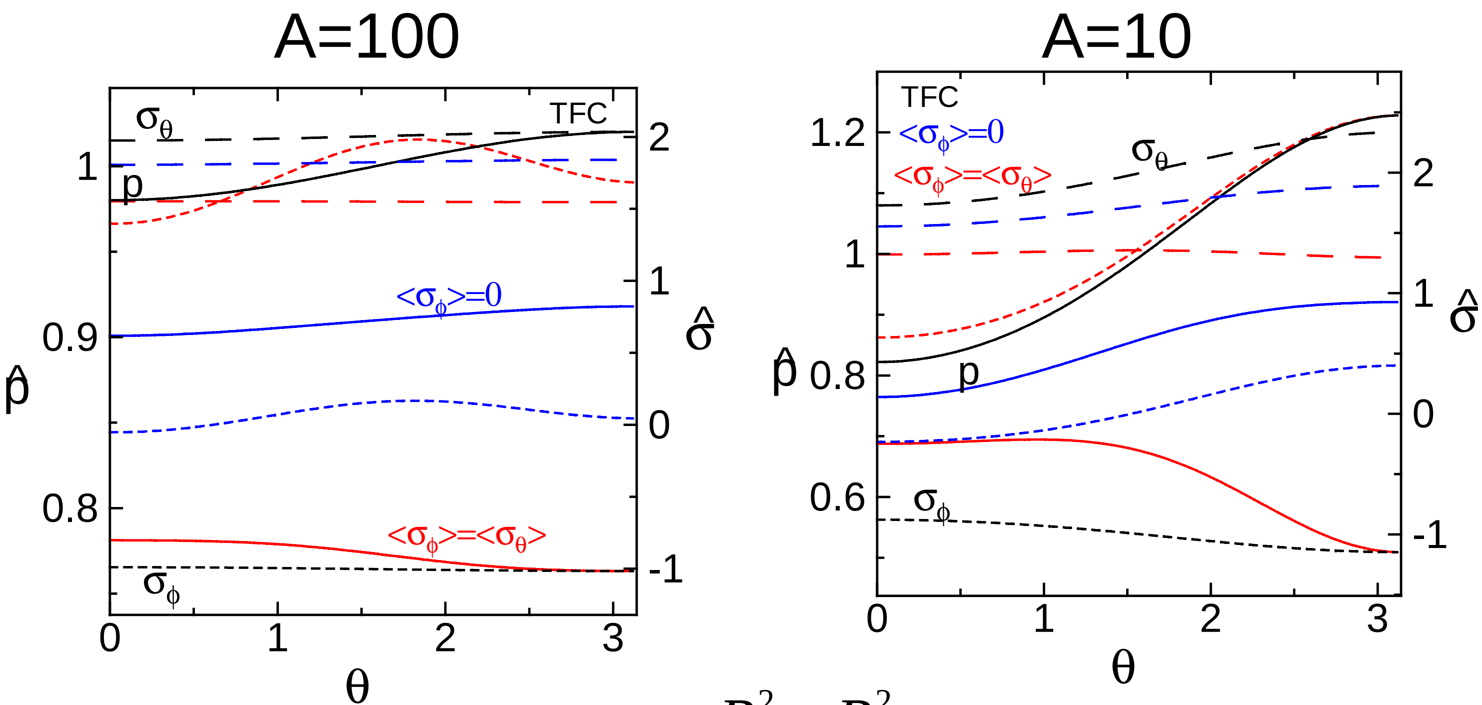
$$T_\theta = \frac{u}{(r-R)r}$$

$$T_\phi = \frac{arp}{r-R} - \frac{u}{(r-R)^2}$$

- Distribution of stress in the toroidal shell with circular cross section is derived analytically by use of magnetic pressure.

Distribution of Stress

(large aspect ratio)

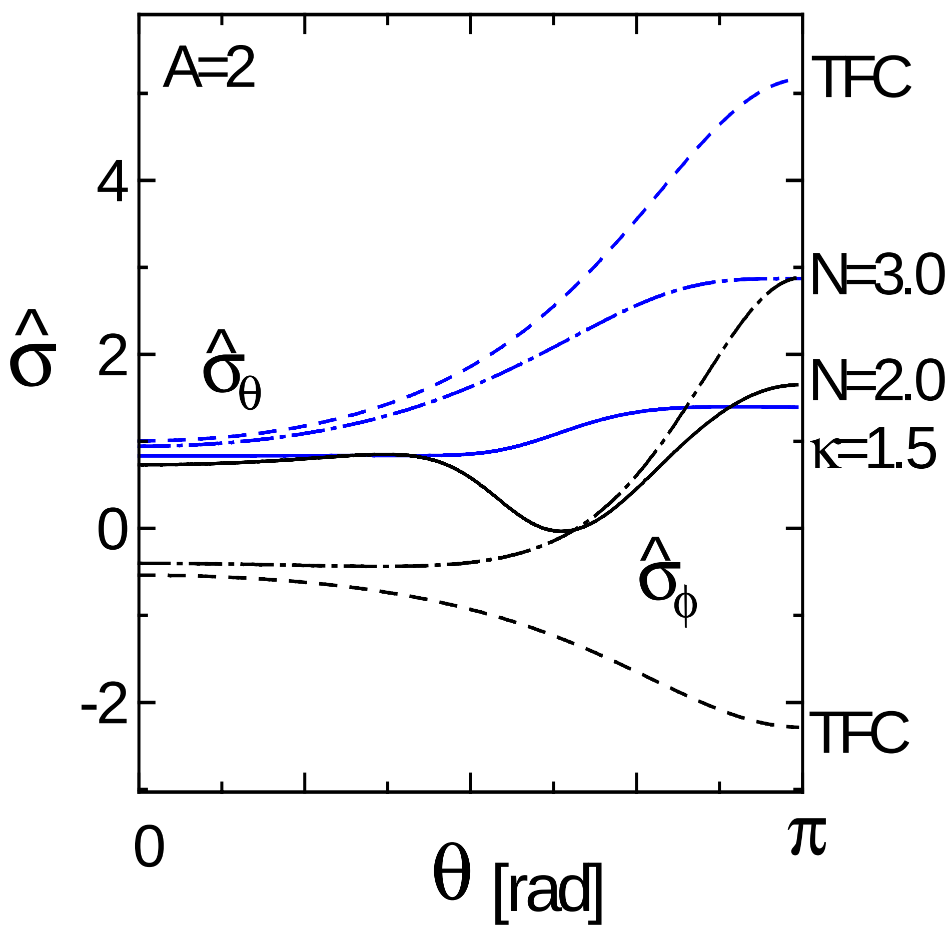


$$\hat{p} \equiv \frac{V_\mathrm{T}}{U_\mathrm{M}} \frac{B_\phi^2 - B_\theta^2}{2\mu_0}$$

- When *A*=100, distribution of stress is flat.
- There is no advantage of helical winding.

Distribution of Stress

(low aspect ratio)



- When A<10, distribution of stress is important.
- Assumption of large aspect ratio is not held.
- Optimal distribution is achieved to minimize the stress at $\theta=\pi$.

Uniaxial Stress Model

Equilibrium of Electromagnetic Force and Stress

$$\frac{\mathrm{d}T}{\mathrm{d}s} + \frac{F_u}{R} = 0$$

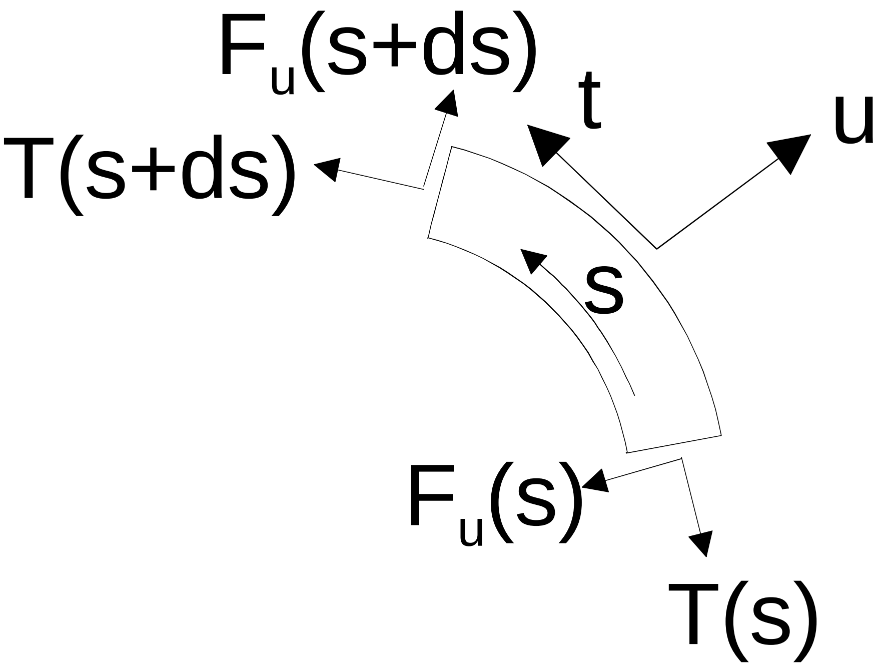
$$\frac{\mathrm{d}F_u}{\mathrm{d}s} + \frac{T}{R} + f_u = 0$$

T: tension, F: sharing force

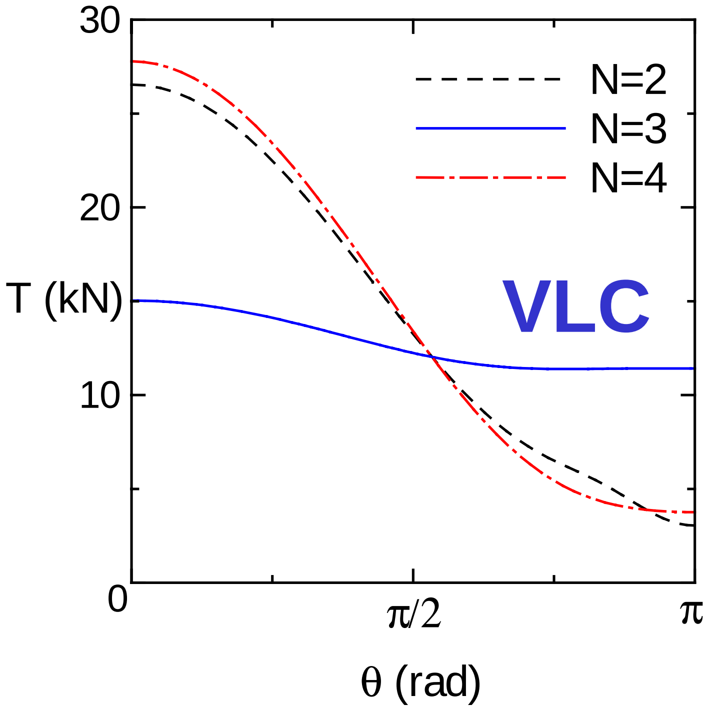
R: curvatur radius,

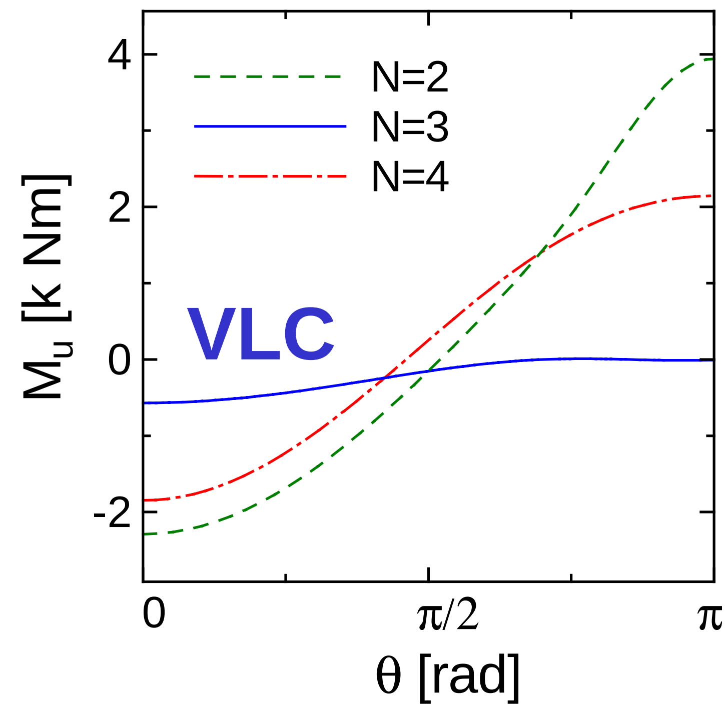
f: electromagnetic force

s: coordinate by length of coil



Tension and Bending Moment





- The tension of coil with pitch=3 is reduced and its distribution is flat.
- In the fat cable, the bending stress (proportional to bending moment) is important.
- The distribution of bending moment in the coil with pitch 3 is flat.

Stress Analysis of VLC

Why FEM analysis ?

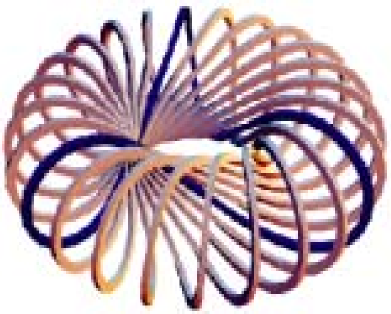
3D analysis is required because the virial-limit condition is obtained from the 2D shell model.

Conditions in FEM Analysis

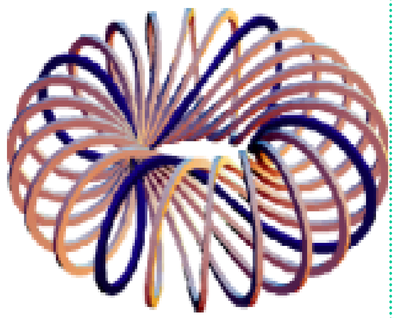
- 3D-Model with Electromagnetic Force
- Structure Analysis by FEM (NASTRAN) → Stress

Parameters	Value
Major radius	0.30 m
Minor radius	0.14 m
Aspect ratio	2.14
Pitch number *N*	24 turns
Coil current	96 kA/1 pole
Toroidal field	1.5 T
Cross section	380 mm^2
Young's modulus	1.26×10^5 N/mm^2
Poisson's ratio	0.33

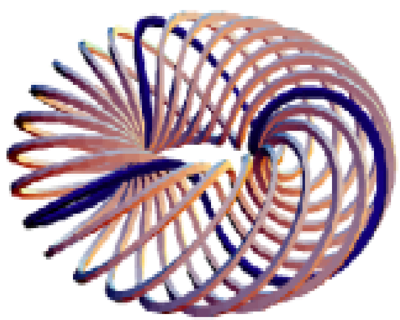
Current layer coincide with magnetic surface.



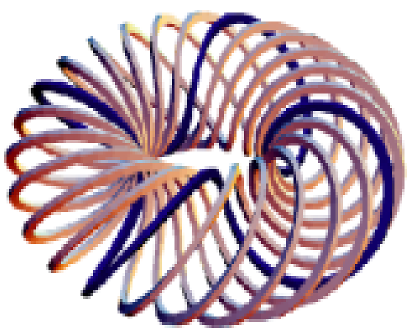
VLC with *N*=3



FBC with *N*=4



HC with *N*=3



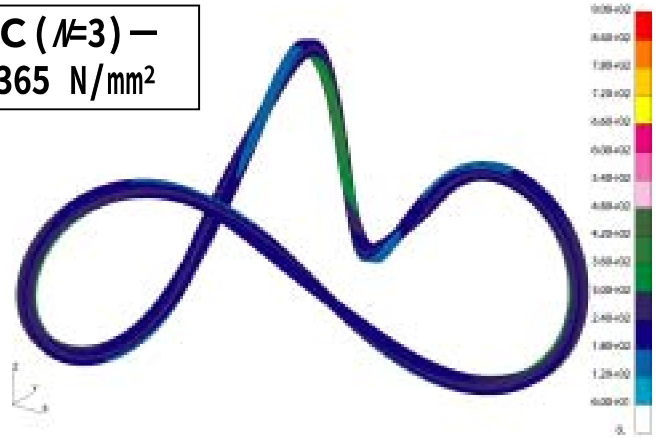
HC with *N*=4

Models in Analysis

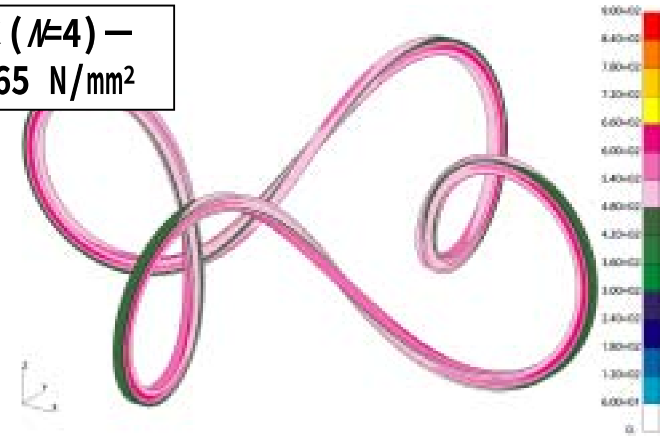
Stress Analysis of VLC

Distributions of von Mises Stress

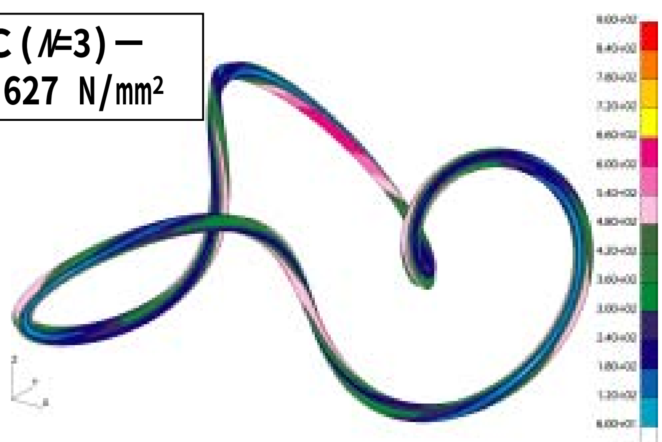
－VLC(N=3)－
Max：365 N/mm²



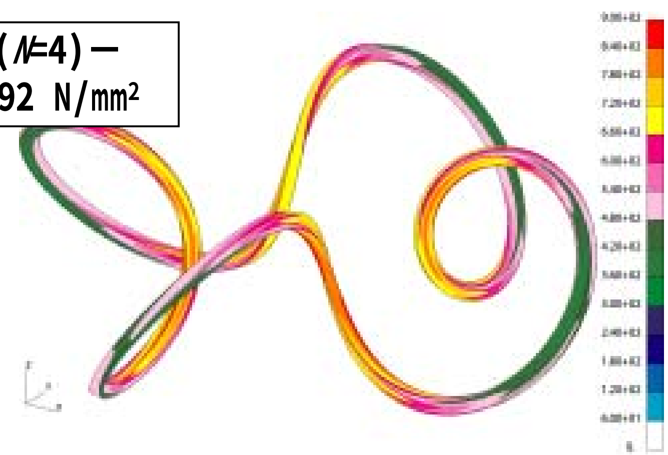
－FBC(N=4)－
Max：665 N/mm²



－HC(N=3)－
Max：627 N/mm²



－HC(N=4)－
Max：892 N/mm²



VLC(N=3) has no stress concentration and minimum stress compared with those of other coils.

VLC realize both nearly uniform distribution of stress and minimum stress in 3D model.

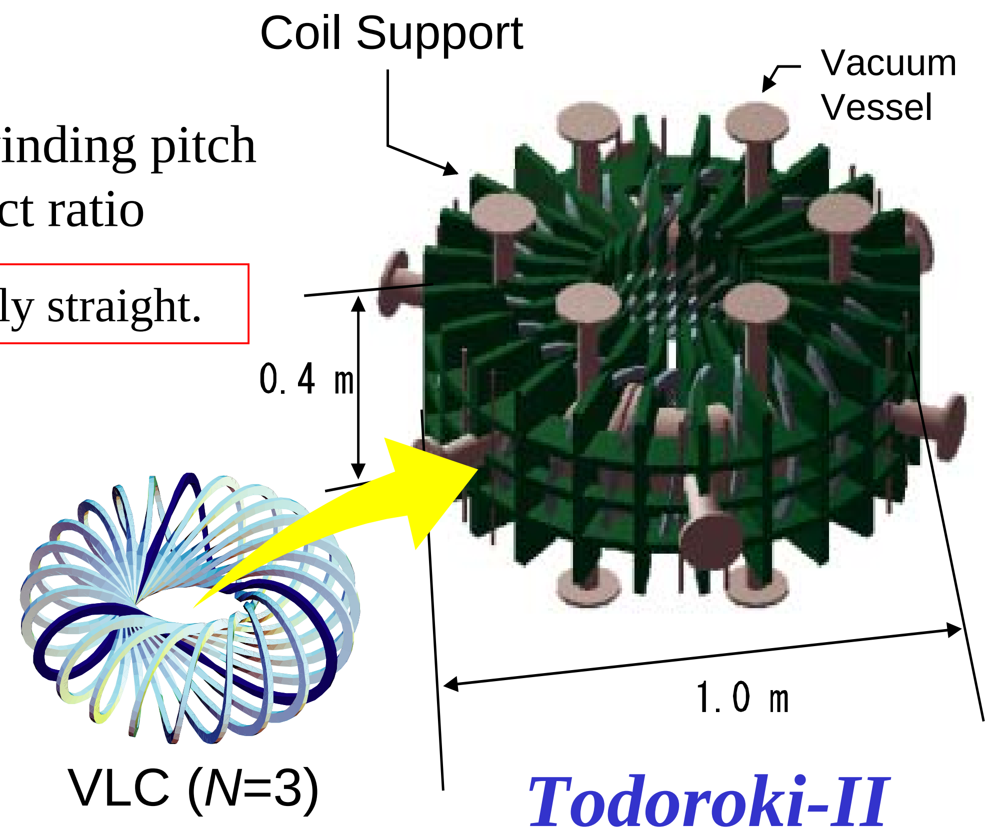
Outline of VLC Tokamak

Outline of Design

Easy winding of coil → integer winding pitch

Large Plasma Volume → low aspect ratio

Coil orbit between their supports is nearly straight.



Parameters of Tokamak

Parameter	Value
Major radius(coil)	0.30 m
Minor radius(coil)	0.14 m
Aspect ratio	2.14
Pitch number	3
Pole number	8
inductance	1.3 mH
Major radius(vessel)	0.30 m
Minor radius(vessel)	0.08 m
Toroidal field	1.55 T
Minor radius(plasma)	0.07 m
Plasma current	40.0 kA

Materials

Coil:	high-tension Kevlar cross section of copper 12.72 mm^2
Vacuum Vessel:	SUS304
Coil Support:	GFRP (thickness 20 mm)

Vertical Field

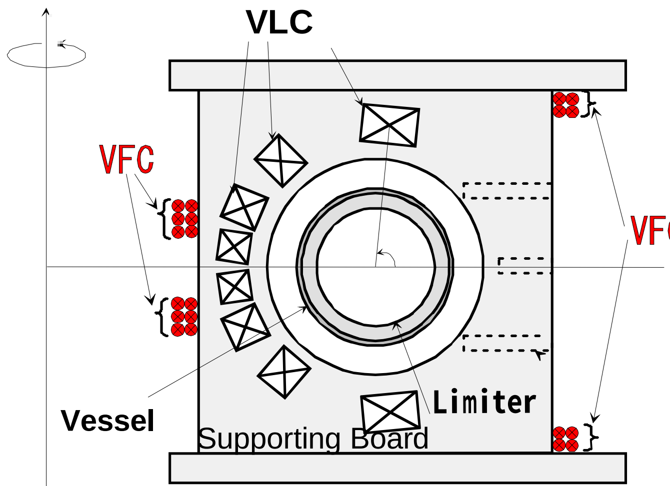
How to design Vertical Field Coil (VFC)

Controllability

→ minimization of mutual inductances to VLC.

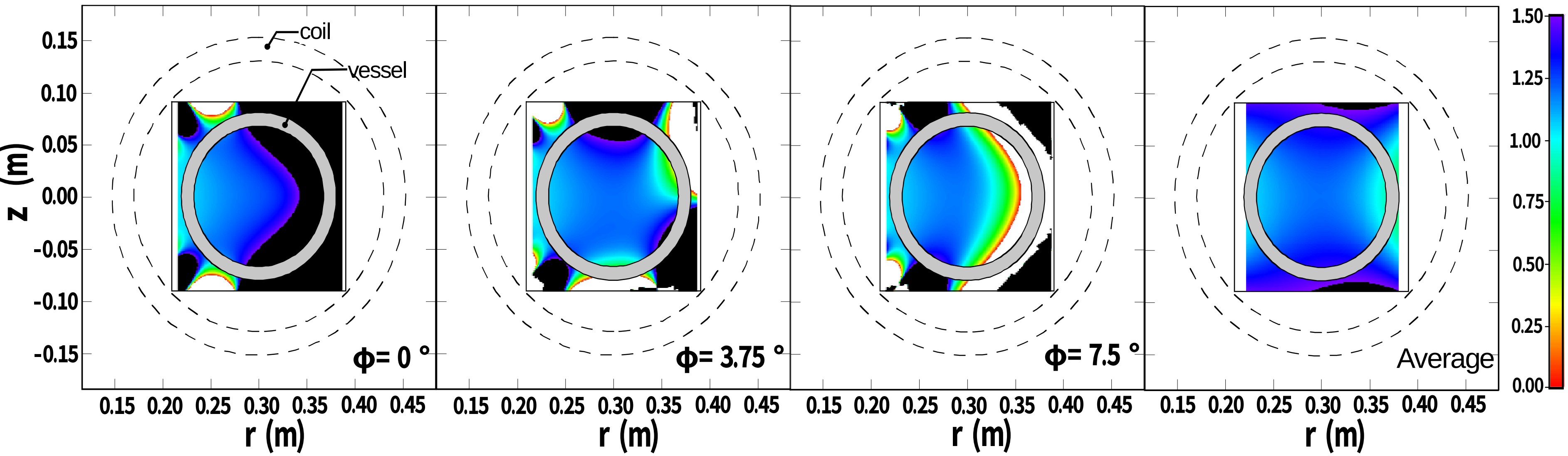
Positional Instability

→ n-index: n (stable condition: 0<n<1.5)



Positions of VFC on the cross section

n-index



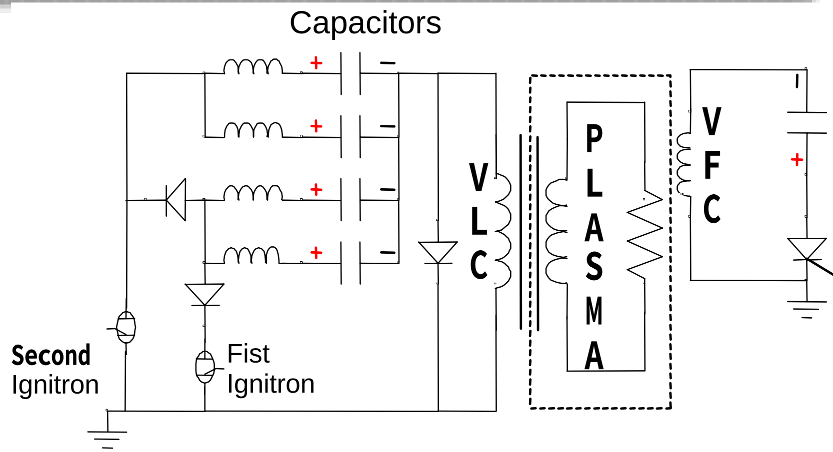
Stable in almost all region in the vacuum vessel

Power Supply and Its Operation

Power Supply

Capacitor x 8	Capacitance :0.5 mF Max Voltage:12.5 kV
Inductance of VLC	1.3 mH

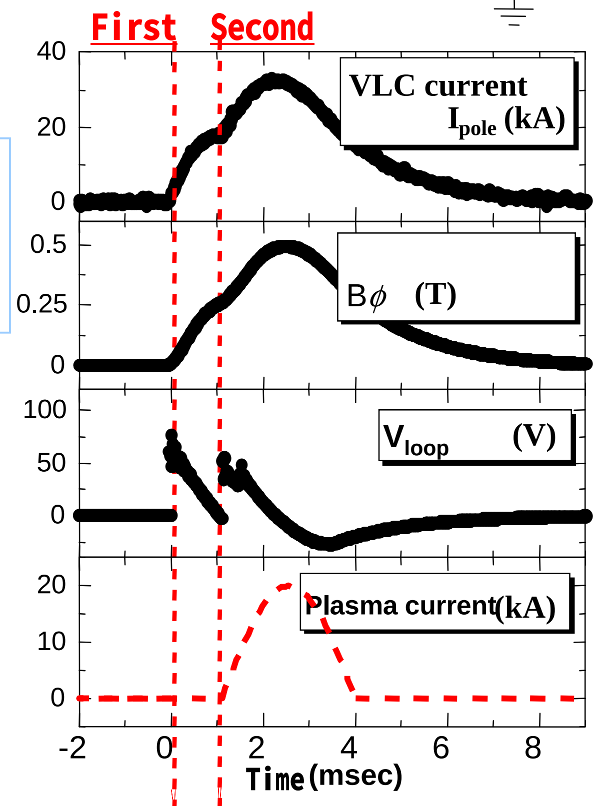
- Discharge period: about 10 msec



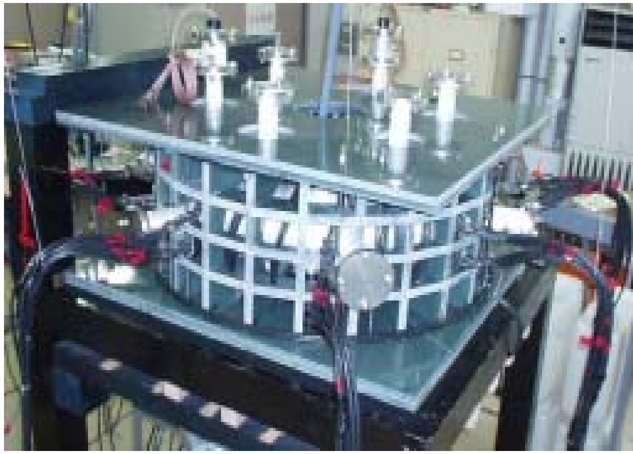
Operation

In the initial stage, there is no toroidal field because VLC is a hybrid coil of CS and TF coil.

First: Toroidal Field
Second: Loop Voltage



Todoroki-II



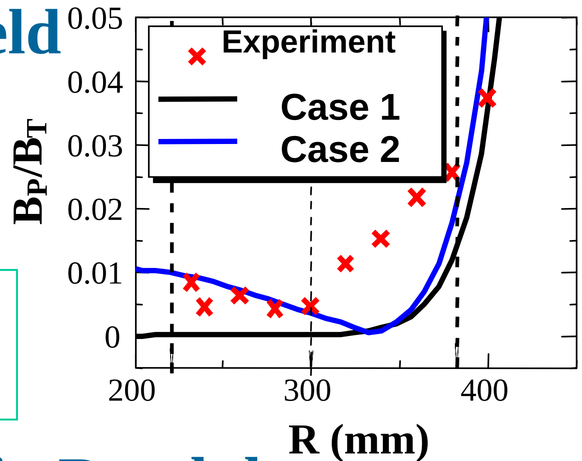
Breakdown Condition

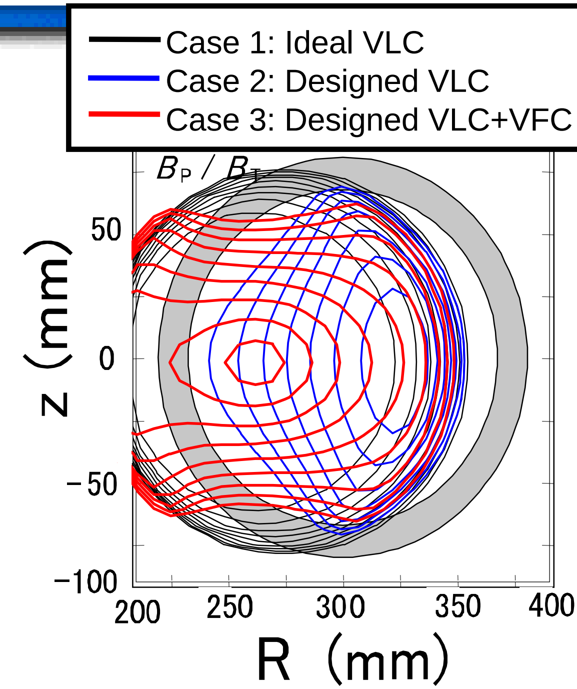
Normalized Poloidal Field

$$B_P / B_T = \sqrt{B_R^2 + B_Z^2} \Big/ B_T$$

Case 1: Ideal Coil Orbit

Case 2: Designed Coil Orbit



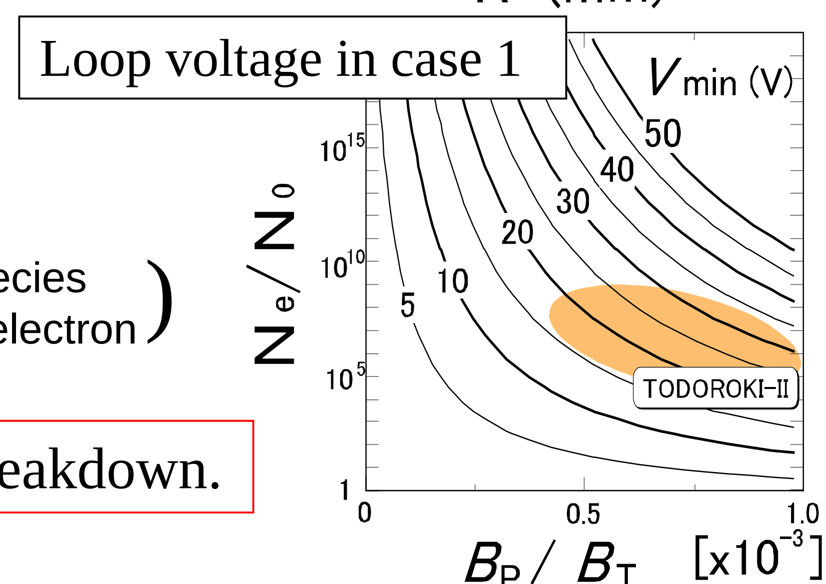


Required Loop Voltage in Breakdown

Next equation is obtained from Townsend Avalanche Model.

$$V_{\min} = 2\pi R_0 \frac{eA_2}{A_1} \log(N_e / N_0) \left(\frac{B_p}{B_T} \right) \Big/ X_P$$

(R_0 : major radius A_1, A_2 : constants of gas species
X_p : limiter radius n_e / n_0 : multiplication factor of electron)



Additional vertical field is required for breakdown.

Summary

- The relation of toroidal field and stress is obtained by **virial theorem**, which shows that the optimal stress configuration is uniform tensile stress.
- When A=2 and κ=2, a virial-limit coil (**VLC**) makes 1.7 times stronger magnetic field than TF coil.
- **VLC** winding generates small error fields, and makes room for blanket and other parts in conventional tokamak reactors.
- Nearly uniform stress distribution with **VLC** configuration is obtained from both uniaxial model and FEM analysis.
- A small **VLC** tokamak ***Todoroki-II*** was constructed and its experiments started.